Abstract: This study examined the dynamics of tidal propagation inside a tropical lagoon. Sea surface elevation (inside) and current profiles (at the inlet) were examined over 60 days at the Chelem lagoon, which is a branched tropical lagoon located in the northern Yucatan Peninsula. Tides were predominantly diurnal with a wavelength at least 20 times longer than the total length of the basin. Spatial variations of sea surface elevation and the longitudinal transport were described in each branch by applying a linear analytical model and the results were compared to observations. Results showed that the coastal lagoon was highly frictional. The tidal signal was attenuated between 30% and 40% toward the lagoon heads, a result of the balance between pressure gradient and frictional forces. A causeway that chokes the western side of the lagoon allowed the propagation of the diurnal signal toward the west head of the basin but damped the semidiurnal signal. The causeway acted as a hydraulic low-pass filter, as in natural choked systems. The causeway's filter effect was included in the analytical model by optimizing the frictional parameters.
November 1, 2015
Ensenada, Baja California, Mexico

Professor Takeshi Matsuno
Associate Editor
Continental Shelf Research

Dear Professor Takeshi Matsuno:

We are resubmitting our manuscript “Tidal variations in a frictionally driven tropical lagoon” by Tenorio-Fernandez Leonardo, Gomez-Valdes Jose, Marino-Tapia Ismael, Enriquez Cecilia, Valle-Levinson Arnoldo, Parra Sabrina M. for publication in Continental Shelf Research.

This manuscript is a revised version of the Ms. Ref CSR3436R3 “Tidal variations in a highly frictional tropical lagoon”. The recommendations from the reviewers are included in the revised manuscript.

The revised manuscript is original, neither it has been published elsewhere nor it has been submitted to another journal.

Enclosed you will find the response to your comments. The response to the reviewer is provided in a separated file. If there is any additional information that you would like to have about our revised manuscript, please do not hesitate to let us know.

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Sincerely,

Leonardo Tenorio-Fernandez
Dear Dr. Gomez-Valdes,

We have received a review for your revised manuscript. I had invited just one reviewer because the other reviewer had suggested "acceptable in this form" for CSR3436R2. Reviewer #1 evaluates that the manuscript has been improved, but still suggests moderate revision with some comments and questions.

The reviewer's comments are not only for the revised parts but also new comments for the previous version. While addressing the reviewer's comments must improve the manuscript and making it easy reading, some of them would be authors' responsibility. For example, the authors referred the formulation of Winant (2007), and it might be better for readers' easy understanding to explain the more details about the formulation as the reviewer pointed out. But it is acceptable if the origin of the formulation or solutions is shown clearly. I just suggest it is better to give explanations for readers to follow them easily. Although I recommend moderate revision again, I will, in some cases, make a decision based on the next revised version without sending the revised version to reviewer. The authors could give just replies to the reviewer's comments, in some cases, instead of revise the manuscript, while revisions may be necessary in the other cases.

Sincerely,

Associate Editor, Takeshi Matsuno

Response:

We would like to express our gratitude to the Editor and the Reviewer for the review process. The authors have followed the suggestions, comments and recommendations from the Editor and Reviewers through the peer-review process; all of them have been addressed even though some of them are repeated in some cases.

The authors consider that it is unnecessary to reproduce parts of the Winant (2007) formulation, since the model has been published in three papers (Winant, 2007; Waterhouse et al., 2011; Henrie and Valle-Levinson, 2014) and the reader may consult these for more information about the mathematical formulation. However, to address your suggestion to explain the formulation and solutions better, the presentation was extended, particularly when referring to the $M_0$, $P_0$, and $Q_0$ parameters and in the model formulation from the non-linear to the linear version.
(these improvements are reflected in Section 4.1). Furthermore, the manuscript was improved in the explanation of the γ parameter, in the longitudinal transport equation origin and in the Henrie and Valle-Levinson (2014)’s methodology. Also it has added a sensitivity analysis of γ for each zone as well as estimates for the eddy viscosity.

The model of Winant (2007) has been used for tidal propagation studies in gulfs (e.g., Adriatic or California) and in long estuaries (Waterhouse et al., 2011; Henrie and Valle-Levinson, 2014). Our implementation of the model to Chelem showed that the combination of the studies by Winant (2007) and Henrie & Valle-Levinson (2014) provide a new methodology for the analytical description of tidal propagation in shallow water bodies. Our research found that tidal currents were driven by the barotropic pressure gradient force acting across a short distance. Chelem was found to be a highly frictional coastal body of water where the tidal signal is attenuated as it propagated within the lagoon because its geometric characteristics contributed to the high frictional effects; furthermore, the tidal signal was strongly attenuated toward the west head because the causeway acted as a hydraulic low-pass filter.

We trust that we have addressed the reviewer’s comments satisfactorily. We believe that the main contentious issue has to do with the presentation of the analytical model, which has been done by other authors. We have, nonetheless, presented more details on the model and results, which should satisfy the misgivings that may have existed.
Reviewer #1:

Reviewer's comments in italic and blue

Review report for CSR3436R3, Revised manuscript dated Sept. 2015.

“Tidal dynamics in a frictionally dominated tropical lagoon.”

by Tenorio-Fernandez et. al.

**Conclusion:** Through the three time revisions, the manuscript was fairly improved, but its story was not straightforward, and confused because of the insufficient explanation. Authors tried to use the 3 dimensional bay oscillation model \((u, v, w)\) first, and next 2 dimensional model by parameterization, and finally 1 dimensional model of dissipative oscillation network. But the key parameters were not clearly stated and the values were not shown. In some cases, the definitions were lacked. Additionally the explanation of modeling results was too rough to suspect. Therefore Reviewer concludes the further revision is requested.

**Response:**

Winant (2007) model has been used in tidal propagation studies in gulfs (e.g., Adriatic Sea and Gulf of California) and in long estuaries (Waterhouse et al., 2011; Henrie and Valle-Levinson, 2014). Our results for Chelem lagoon showed the wide range of applications of the model. In this revision, we try to focus on the explanation of the analytical model. Winant (2007) model is published in the Journal of Physical of Oceanography, a very-well known journal, such that it is not
necessary to reproduce portions of mathematical formulations. However we have extended the model explanation. We still believe that the model has been described in at least those 3 publications and therefore has already been presented in sufficient detail.

**Comment:**

1) Page 11: Authors referred the Winant (2007) and introduced the formula?

\[ N^{(0)} = \frac{\cos[k\mu(1-x)]}{\cos(k\mu)}, \]  

(4)

and

\[ M_o = (f^2 Q_o^2 / P_o) - P_o, \]  

(5)

Reviewer considers (4) is resemble of simple bay oscillation mode \( N^{(0)} = \cos[k(L-x)]/\cos(kL) \) (L: wave length), and \( M_o \) and \( \mu \) may relate the non-dimensional wave length. More detailed explanation of (4) and \( M_o \) are necessary because the dissipative oscillations (7)-(10) are extended from (4) and \( M_o \) is used in (15). Definition and explanation of \( P_o \) and \( Q_o \) are lacked. Even if they were defined in Winant (2007) in detail, the clear definition and brief explanation should be written in this manuscript.

**Response:**

The analytical model explanation was extended with minor alterations, particularly when referring to the \( M_o, P_o, \) and \( Q_o \) terms. Additionally, we have detailed the
model simplification from non-linear to linear. Part of the description in Section 4.1 has been improved, for example (please see pages: 10 last paragraph, 11 last and 12 first paragraph):

Page 10: Because $\varepsilon$ was assumed to be small, the advective terms in the original momentum equations were neglected and the problem was broken into an ordered set of linear problems for the dependent variables. The resulting linear problem in the non-dimensional form to lowest order depended on four parameters $\alpha$, $\delta$, $\kappa$, and $f$.

Page 12: In the equation (4) the frictional effects were represented by the parameter $\mu = \langle M_o \rangle^{-1/2}$, where the angled brackets show the cross-sectional average of $M_o$, and $x$ was the longitudinal position. In turn, $M_o$ was given by

$$M_o = (f^2 Q_o^2 / P_0) - P_0,$$

(5)

where $P_0$ and $Q_0$ are complex functions of $(h, f, \delta)$. Depth was represented by $h$ and only depended on the $y$-axis ($y$ was the transverse-channel direction at all the zones). The $P_0$ and $Q_0$ equations were trigonometric complex functions of three terms; in both cases the first term was independent of $\delta$ (Winant, 2007). Winant showed that when $\delta \to 0$, $M_o$ tended to $h$ and the real part of $\mu = \langle h \rangle^{-1/2}$ was independent of $f$, on the other hand when $\delta \to \infty$, $M_o$ tended to $-P_0$ and also it was independent of $f$. Thus he demonstrated that the low order solution (equation 4) was practically not affected by rotation, which implies that the $f$ term in the momentum equations is negligible.

Note: It is worth noting that the dissipative oscillation (7)-(10) is not extended from (4). The equation (4) was an approximation used only for diagnostics and for
frictional classification. Knowing that Chelem is highly frictional and following Winant (2007), equation (4) was not used because the wave is damped toward the closed end. Instead, the solution for tidal longitudinal propagation \( N^{(0)} \) in highly frictional water bodies becomes equation (7). We’ve done this despite the fact that these explanations have been presented in the three papers mentioned above (Winant, 2007; Waterhouose et al., 2011; Henrie & Valle-Levinson, 2014).

Comment:

2) Page 14: Explanation of \( \gamma \) is insufficient, please add more detail

Response:

The explanation of \( \gamma \) was added with more detail at the end of page 14, as follows:

The \( \gamma \) parameter depends on the geometry of the basin and is of order 1 (Winant, 2007). To select the optimal values of \( \gamma \) at each Zone, sensitivity analysis between 0.2 and 1.2 were performed. The best fit between observations and the model results gave optimum values of \( \gamma \). For Zone 1 (short channel) lower values of \( \gamma \) (~0.2) were used and for Zones 2 and 3 (long channels) intermediate values (~0.7) were used.

Comment:

3) Page 15: Authors suddenly show the formula (15),
This formula is very important to determine how to divide the oscillation to each zone satisfying the condition of continuity in the junction of zone 1, 2 and 3. So more detail explanation and origination of (15) are necessary.

Response:

The channel connections explanation was improved. Please see the last part of Section 4.3. An extended explanation was added on the origin of the longitudinal transport equation at each zone:

When the Coriolis term was negligible, the lower order longitudinal local velocity solution $U$ was function of $N_x^{(0)}, p_0,$ and $k$, where $p_0$ was a complex function of $h, z$ and $\delta$. The longitudinal local velocity was depth-integrated to obtain horizontal transport in the longitudinal direction.

Comment:

4) Page 19: Error "The $M_2$ showed a small negative semi-minor axis; however this result was not reliable because the error associated to the semi-minor (semi-major) axis was of similar magnitude as the semi-minor axis value.

Response:

Please see the second paragraph of page 20.
**Comment:**

5) Page 21: Authors wrote “A scaling analysis was performed between three non-linear terms (the advective \( u \partial u / \partial x \) and the friction terms \( \eta |u| / h^2 \) and \( u |u| / h \)) to demonstrate their relative importance.” The first advective term and third frictional term are understandable but what is the second frictional term? Reviewer considers the first and third terms are appeared in original tidal component equation, but the second term in compound or over tidal component. So they should not be compared in here, is it correct?

**Response:**

It is partially correct. It depends on the expansion of the non-linear terms. However, the description of the non-linear terms analysis has been shortened because the third term was the most important (please see the second paragraph of page 22), as follows:

A scaling analysis was performed between the nonlinear terms (the advective \( u \partial u / \partial x \) and the frictional term \( u |u| / h \)) to demonstrate their relative importance.

The magnitude of the tidal velocities \( (u) \) were order \( 10^{-1} \) with a longitudinal scale, \( x \), of order \( 1 \times 10^3 \) and the depth \( (h) \) of order 1. Therefore the magnitude of \( 1/h |u| \) was at least two orders of magnitude larger than the other term of the one-dimensional momentum equation \( (1/h |u| \gg u \partial u / \partial x) \). Furthermore, since the
study area is a highly frictional lagoon, the balance between the pressure gradient and frictional forces controls it. Following Hill (1994), the balance was written as

\[ g \frac{\eta_0 - \eta_L}{L} = \frac{C_f u|u|}{(H + \eta_m)}, \]

where the sea surface elevation in the open end was \( \eta_0 \), the sea surface elevation in the lagoon was \( \eta_L \), \( \eta_m \) was the mean surface elevation between them, \( C_f \) was the dimensionless friction coefficient. Consequently the quadratic part of the non-linear friction term \( \frac{1}{h} u|u| \) was expected as shallow water tidal generator.

**Comment:**

6) Page 23: Authors wrote "Once the \( \delta \) and \( \kappa \) parameters were determined for each zone and for each tidal harmonic of interest. The Fig. 7a only shows the and parametric space for the diurnal frequency in Zone 2 as an example, but the same analysis was performed for each zone and frequency." \( \delta = (2K^*/\omega^*H^*)^{\frac{1}{2}} \) and \( \kappa = \omega^*l^*/(g^*H^*)^{\frac{1}{2}} \), as \( \omega^* \) is fixed as tidal frequency, and eddy viscosity \( K^* \) is determined, so that \( \delta \) and \( \kappa \) are fixed. Why the optimal fitting of the least square error in \( \delta \) and \( \kappa \) parametric space is necessary? Reviewer considers the solution in (7) and (8)-(10) is not fixed only by \( \delta \) and \( \kappa \), but \( M_0 \) in (15) including \( \delta \), is indispensable, so that the optimal fitting may be included for \( M_0 \) validation, is it correct? In anyway, what value is adopted for \( K^* \)?

**Response:**
Thank you for your comment. To improve the description in these aspects, the following sections were modified: Section 4.1 Model description (last paragraph of the section), the second paragraph of section 4.2 Analytical solution (Henrie and Valle-Levinson (2014)’s methodology) and the first paragraph of the section 5.4 Model results (eddy viscosity calculations).

The answers of the specific reviewer questions are:

- The optimal fitting of the least square error in $\delta$ and $\kappa$ parametric space is necessary because $\delta$ is unknown.
- $M_0$ is a function of $\delta$. Therefore if $\delta$ is known, $M_0$ is also known.
- From $\delta$ optimal values, the eddy viscosities were calculated to each zone and frequency. The vertical eddy viscosities were approximately 0.02 m$^2$/s at Zones 1 and 3, this value is similar to that found by Munk (1966), at Zone 2 the vertical eddy viscosities were approximately 0.003 m$^2$/s, coinciding with the values reported by Waterhouse et al. (2011) for this type of channel.

**Comment:**

7) Figure 4: For diurnal band, there are 2 change points of 0.037 and 0.042 cph in Gain graph, but 1 point of 0.042 cph in Phase graph, why? For semidiurnal band, the point of 0.08 cph may be a little bit out of position in Gain and Phase, why?

**Response:**

Thank you for your observation. The gain and phase lag diagrams have been drawn again. The gain and phase diagram have the same change points. The
position of the result points are the same. Furthermore the interpretation of Figure 4 is presented on page 18.

Comment:
8) Figure 7 (c): Why diurnal phase in zone west (blue + mark) is so out of position?
Other marks in figure 7(b) and (c) are fairly in position. Authors wrote in page 24, "The analytical time delay diverged from the time delay observed (around 3.2 h) at the end of Zone 2 (west head) especially with the diurnal signal, while those of Zone 1 (0.1 h) and Zone 3 (0.6 h) were consistent (Fig. 7c). ", and for the reason, "The phase differences were because the model did not consider the causeway's effect directly. ". This reason is too poor to explain the model calculation result.
Please investigate the reason further and explain carefully.

Response:
Thank you for your comment, the description of the analytical time delay diverged from the time delay observed has been extended, as follows:
The analytical model describes the linear problem. The fit between observations and the model results was suitable except in the causeway zone where the advection terms are especially significant due to the narrow channel effects. The effect of these model limitations was more noticeable at the Zone 2 especially in the phase differences between observations and the phase model results.
The narrow channel effect of the causeway’s gap produces turbulence at the gap and adjacent areas. The Reynolds number was used to quantify the importance of the non-linear terms. The Reynolds number is defined as:

$$R_e = \frac{UL}{K},$$

where $U$, $L$ and $K$ were the velocity scale, the scale length and the eddy viscosity, respectively. The velocity scale was calculated using the analytical longitudinal transport at the end of the Zone 1 for the principal tidal component (~60 m$^3$/s) and the gap transversal area (~30 m$^2$). The length of the narrow channel effects was considered as 100 m. In Zone 2 eddy viscosity was 0.003 m$^2$/s. The dimensionless Reynolds number for $K_1$ was

$$R_e = \frac{UL}{K} \sim \frac{(2 \text{ m/s})(100 \text{ m})}{0.003 \text{ m}^2/\text{s}} \sim 60000$$

And for the principal semidiurnal component $M_2$ was

$$R_e = \frac{UL}{K} \sim \frac{(1 \text{ m/s})(100 \text{ m})}{0.003 \text{ m}^2/\text{s}} \sim 30000$$

Therefore, the flow at the narrow channel and adjacent area is highly turbulent. It occurs only in this part of the lagoon, but the effect was noticeable in the phase differences between observations and the phase model results at the Zone 2.
Tidal propagation inside a semi-closed branched system was examined.

The *Chelem* lagoon was determined to be a highly frictional basin.

The momentum balance in *Chelem* was between pressure gradient and frictional forces.

The tidal signal was most attenuated toward the west head due to a causeway.

A fortnightly-period fluctuation related to the tropical spring tide was observed.
Tidal dynamics in a frictionally dominated tropical lagoon

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Re-submitted to
Continental Shelf Research

October-September 2015
Abstract

This study examined the dynamics of tidal propagation inside a tropical lagoon. Sea surface elevation (inside) and current profiles (at the inlet) were examined over 60 days at the Chelem lagoon, which is a branched tropical lagoon located in the northern Yucatan Peninsula. Tides were predominantly diurnal with a wavelength at least 20 times longer than the total length of the basin. Spatial variations of sea surface elevation and the longitudinal transport were described in each branch by applying a linear analytical model and the results were compared to observations. Results showed that the coastal lagoon was highly frictional. The tidal signal was attenuated between 30% and 40% toward the lagoon heads, a result of the balance between pressure gradient and frictional forces. A causeway that chokes the western side of the lagoon allowed the propagation of the diurnal signal toward the west head of the basin but damped the semidiurnal signal. The causeway acted as a hydraulic low-pass filter, as in natural choked systems. The causeway's filter effect was included in the analytical model by optimizing the frictional parameters.

Keywords: Choked coastal lagoon; tidal hydrodynamics; highly frictional; Chelem lagoon.
1. Introduction

Coastal lagoon hydrodynamics are influenced mainly by tides, winds, heat fluxes, and freshwater inputs. The dynamics of coastal lagoons are also shaped by inlet morphology. Coastal lagoons may be subdivided according to their inlet characteristics into leaky, restricted and choked systems (Kjerfve and Magill, 1989). Leaky lagoons are longest and narrowest ($\sim 10^3$ and $\sim 10^2$ m, respectively), parallel to the coastline and their hydrodynamics are controlled directly by the ocean through many inlets. Restricted systems are typically oriented parallel to the coastline, have one or two inlets and a well-defined tidal circulation, where the dynamics are controlled by the adjacent ocean. Choked lagoons are usually found along high wave energy coastlines with marked littoral drift and have one or more narrow inlets. Dominant wind forcing and freshwater pulses that may produce intermittent vertical stratification characterize these lagoons. In choked systems, the tidal signal is altered or eliminated because the inlet acts as a dynamic low-pass filter (Kjerfve and Magill, 1989). Choked systems, especially those located in the tropics, are a very important part of the local ecology owing to their high primary and secondary production (Krumbein et al., 1981; Barnes, 1980). Therefore, understanding their dynamics is essential because they play an important ecologic and economic role (Albrecht and Vennell, 2007).

In general, the hydrodynamics of choked lagoons are poorly understood. Such is the case in most of the lagoons that line the Mexican coast of the Gulf of Mexico (GoM). Considering this dearth of knowledge in the tropics, the approaches that
have been used in subtropical and temperate regions can also be applied to study these systems. The purpose of this investigation is to analyze the dynamics of tidal propagation inside a branched tropical lagoon with predominant diurnal tides. The tidal propagation dynamics of the lagoon were analyzed using a linear model proposed by Winant (2007). This study optimized different frictional and geometric parameters on the basis of observations in the lagoon, following the approach by Henrie and Valle-Levinson (2014). Analyses of the data and of the analytical model output provided the frictional characteristics of the two branches of the lagoon.

2. Study area

The Chelem lagoon is located between 21°10’ and 21°19’N and between 89°47’ and 89°37’ W (Fig. 1b). It is a branched tropical lagoon, which extends parallel to the coast in an E-W orientation, and is shallow with depths ranging from 0.7 m to 3.5 m. Since 1969, many physical modifications have affected Chelem. The most drastic alteration was the construction of a causeway across the lagoon in the 1980s. This causeway divided the lagoon and severely restricted water circulation toward the western head. The causeway has two bridges, each one with 5 m-wide gap that allows some water flow and maintains the west head connected to the central lagoon. However, this restriction modifies the natural hydrodynamics (e.g., see Hill, 1994) and has already caused diverse environmental problems.

The lagoon is a network of three zones connected by the central lagoon area (Fig. 1b). Zone 1 is a meridional channel that connects the lagoon with the Yucatan
Shelf. This Zone is the deepest, narrowest, and shortest (Table 1 shows the average dimensions of each zone). Zones 2 and 3 bifurcate from Zone 1 and are channels oriented westward and eastward, respectively. Zone 2 is the longest, shallowest, widest and is affected by the causeway. It is connected with Zone 1 only through the causeway gaps. Zone 3 extends from Zone 1 to the eastern head, which has medium length, medium width, and medium depth.

*Chelem* tidal signals related with those of the Yucatan Shelf that are forced by the GoM tides (Fig. 1a). Tides in the GoM are the result of indirect tidal oscillations from the Atlantic Ocean and direct astronomical forcing (Zetler and Jansen, 1972). The main tidal constituents within the GoM are the lunisolar diurnal \( (K_1) \), the lunar diurnal \( (O_1) \), and the principal lunar semidiurnal \( (M_2) \); however, tide behavior varies along the coast (Kantha, 2005). The tides range from semidiurnal near Florida to diurnal at the Yucatan Peninsula according to tidal station information from the National Oceanic and Atmospheric Administration. David and Kjerfve (1998) found that at the *Terminos* lagoon in the neighboring Campeche state, Mexico (~400 km from *Chelem*) the tidal range is 0.3 m with diurnal dominance; this pattern continues all the way to the Yucatan Shelf.

Kjerfve (1981) studied the main tidal amplitudes and tidal phases along the Yucatan Shelf. He found that the amplitudes of the diurnal components \( K_1 \) and \( O_1 \) at the *Progreso* harbor (located ~10 km east of *Chelem*) were 17.7 cm and 17.1 cm, respectively, with a \( M_2 \) as the largest semidiurnal amplitude component; with a value of 6.0 cm. Martínez-López and Pares-Sierra (1998) used a three-dimensional
model to study tidal dynamics in the Yucatan Shelf. However, the tides within Chelem, or any lagoon in Yucatan, have yet to be described.

The tidal signal inside semi-closed coastal basins is modified by friction, the Earth’s rotation and morphology (Waterhouse et al., 2011). Frictional effects dominate over the Earth’s rotation in shallow basins (Winant, 2007). For highly frictional basins, the relation between sea surface elevation and water velocity departs from the classical wave equation solutions (frictionless). Therefore, tidal propagation in these systems could be described using the diffusion equation (LeBlond, 1978; Friedrichs, 2010). This diffusive process suggests that greater friction induces faster tidal amplitude attenuation. Under these conditions, the equation describing the tidal signal propagation is one-dimensional along the basin and is only determined by the pressure gradient and frictional forces, regardless of lateral depth variations along the channel (Li and Valle-Levinson, 1999; Waterhouse et al., 2011). These premises are explored in Chelem with observations and analytical model results.

### 3. Materials and methods

Data used in this study consisted of a series of moored instruments: a current profiler at the mouth and six hydrographic instruments throughout the lagoon. At the entrance to Chelem, a velocity time-series was obtained from a bottom-mounted, upward pointing Aquadopp Doppler current profiler (Fig. 1b and Table 1). The time series spanned from 27 June 2012 to 25 August 2012 (60 days). The
instrument was deployed at the center of the lagoon’s navigation channel where the average depth is 3.5 m with respect to mean sea level. Observations were divided into 10 cells, each of 0.3 m length, with a blanking distance of 0.5 m at the bottom and 0.3 m at the surface (i.e., the last cell); thus, the profiles ranged from 0.5 m to 3.20 m above the bottom. The velocity profiles were obtained every 60 s and saving one averaged value every 15 min. The accuracy of the current profiler is 1% of the measured value, typically ±0.5 cm/s.

To provide a more extensive description of the tides within the lagoon, water level data were obtained from six deployed Schlumberger Diver conductivity-temperature-depth recorders (CTDs). Sampling interval for all Diver CTDs was 10 minutes. The present study concentrated on tides so it only analyzed water level data. One of the Diver CTDs was deployed at the mouth next to the current profiler, and the other five were inside the lagoon (Fig. 1b). The accuracy of the pressure sensor was ± 4.905 × 10⁻⁴ bar (~0.5 cm depth).

At Zone 1, a current profiler and Diver CTD were deployed at the mouth and a Diver CTD at station C1 (Fig. 1b and Table 1). In Zone 2, one Diver CTD was affixed at the west head. In Zone 3, Diver CTDs were moored at C2, C3 (in the middle of the channel) and at the eastern head. High frequencies and noise were removed from sea surface elevation data with a Lanczos filter (cut-off frequency of 0.25 h⁻¹). Power spectrum was calculated for each time series, with a 95% confidence interval (Emery and Thomson, 2001).
The spectral relationship between station C1 signal and the west head signal was evaluated to understand the effects of the causeway on the west channel. The frequency response function \( R = S_{xy}/S_{xx} \) was used to analyze the impact of the causeway, where \( S_{xy} \) is the cross-spectra between station C1 (input signal) and the west head (output signal), and \( S_{xx} \) is the auto-spectrum of station C1 signal, the response function is defined as the spectral gain and the phase between two signals (the input and output). The confidence level of 95 % was calculated using the coherence function, which is defined as \( C^2 = \left| \frac{S_{xy}}{S_{xx}S_{yy}} \right|^2 \), where \( S_{yy} \) is the auto-spectrum of the west head signal. The values of the response function that were below the confidence level, were not considered in the analysis, and for the significant values the error bars were calculated (Bendat and Piersol, 2010).

Harmonic analysis was applied to the time series following Gomez-Valdes et al. (2012). Amplitudes and tidal phases of the main tidal constituents were obtained with the least squares method, which included the Rayleigh criterion of 1 and nodal corrections. The estimated uncertainty of the constituents was calculated with the signal-to-noise ratio \( (snr) \) parameter. The \( snr \) was calculated by \( snr = (M/\delta M)^2 \), where \( M \) is the amplitude of tidal constituents and \( \delta M \) is the amplitude error (Pawlowicz et al., 2002). When \( snr \) is high \( (snr \gg 1) \), the harmonic constituent is well resolved, but when \( snr \) approaches 1 the constituent is unreliable because the uncertainty is of the same magnitude as the estimate.
Data from the current profiler were analyzed in the same way as those for sea surface elevation. Using the along channel velocities at the lagoon entrance, the power spectrum was calculated for each depth bin following Gomez-Valdes et al. (2012). The vertical structure of the currents was studied from the time average of the longitudinal velocity of each bin. The harmonic analysis of the currents was performed on the vertically averaged velocity and on each bin. In both cases, the tidal ellipse parameters (semi-major axis, semi-minor axis, phase, and orientation) were obtained following Pawlowicz et al. (2002).

4. Analytical model

4.1. Model description

The tidal circulation in a branched and choked lagoon was described with a three-dimensional, non-linear, homogeneous (constant water density) model as explained by Winant (2007). The linear, lowest-order solution of the model was used in the analysis. The model has been used in tidal propagation studies in gulfs (Winant, 2007) and in long estuaries (Waterhouse et al., 2011; Henrie and Valle-Levinson, 2014).

The model describes tidal wave propagation through semi-closed elongated water bodies using the relationship between geometric and frictional characteristics, assuming shallow water waves approximation, negligible lateral mixing, constant
vertical eddy viscosities, constant channel width and constant depth in the longitudinal axis. Hereafter dimensional variables are represented with asterisk.

The solution was based on the linearized momentum equations. The set of equations that describe the original non-linear problem for shallow elongated channels depended on five non-dimensional parameters $e, \kappa, \delta, \kappa, \text{ and } f$. The maximum depth of the system $H^*$ was considered much greater than the open-end tidal amplitude $C^*$ ($C^*/H^* \ll 1$). Hereafter dimensional variables are represented with asterisk.

The set of equations that describe the $s = C^*/H^* \ll 1$) linear problem for shallow elongated channels depended on four non-dimensional parameters $\alpha, \delta, \kappa, \text{ and } f$. The horizontal aspect ratio $\alpha$, it was the ratio between width $B^*$ and length $l^*$ of channel, $\alpha = B^*/l^*$. For elongated basins, ($\alpha \ll 1$), the lower order solution predicts zero pressure gradient at the $y$-axis momentum equation. The frictional parameter $\delta$, relates friction to local acceleration (a proxy of the Stokes number), and it was defined as $\delta = (2K^*/\rho^*H^*)^{\frac{1}{2}}$, where $K^*$ stands for the vertical eddy diffusivity and $\omega^*$ for the tidal frequency. The geometrical parameter was $\kappa$. It represents the relative measure between the length of the channel and the wavelength of the tidal wave, it was defined as $\kappa = \omega^*l^*/(g^*H^*)^{\frac{1}{2}}$, where $g^*$ stands for gravitational acceleration. Finally, $f^*$ was the non-dimensional Coriolis parameter it was defined as $f = f^*/\omega^*$, where $f^*$ stands for the Coriolis frequency.
Because \( \varepsilon \) was assumed to be small, the advective terms in the original momentum equations were neglected and the problem was broken into an ordered set of linear problems for the dependent variables. The resulting linear problem in the non-dimensional form to lowest order depended on four parameters \( \alpha, \delta, \kappa, \) and \( f; \) the linearized non-dimensional momentum equations were:

\[
\frac{\partial u}{\partial t} - f\alpha v = -\frac{1}{\kappa^2} \frac{\partial \eta}{\partial x} + \frac{\delta^2}{2} \frac{\partial^2 u}{\partial z^2},
\]

\[
\frac{\partial v}{\partial t} + \frac{f}{\alpha} u = -\frac{1}{\alpha^2 \kappa^2} \frac{\partial \eta}{\partial y} + \frac{\delta^2}{2} \frac{\partial^2 v}{\partial z^2},
\]

and the continuity equation in the form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.
\]

Where the boundary conditions were no slip at the bottom \((z = -h)\) and kinematic and dynamic boundary conditions at the surface \((x = 0)(z = \eta)\). The no-slip condition considered that the velocities were zero at the bottom \((i. e. u = v = w = 0)\). At the surface, the kinematic condition implies that the water particle never leaves the surface \((i. e. w = \frac{\partial \eta}{\partial t} + \varepsilon \left( \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} \right) \) and since \( \varepsilon = C^*/H^* \ll 1 \), the second term on the right hand of the kinematic boundary condition is linearized, producing: \( w = \frac{\partial \eta}{\partial t} \). The dynamic condition indicates that the pressure is constant and there is no shear stress \((i. e. \frac{\partial u}{\partial x} = \frac{\partial v}{\partial z} \frac{\partial u}{\partial x} = v_z = 0)\). Since there was not lateral mixing in the model, there was no need for non-slip and shear stress.
boundary conditions at the lateral boundaries. On the open boundary the tide forces the sea level.

Using periodic solutions to find the corresponding set of differential equations from the vertically integrated continuity equation, Winant (2007) expanded the amplitude \( N \) in function of powers of \( \alpha \) and found a lower order \( \mathfrak{N}^{(0)} \) solution for sea surface elevation that satisfies the boundary conditions. The solution was given by

\[
N^{(0)} = \frac{\cos[k\mu(1 - \chi)]}{\cos(\kappa\mu)}, \quad (4)
\]

This solution was limited to the constant width and the depth on function of \( y \)-axis only. The frictional effects were represented by the parameter \( \mu = \langle M_0 \rangle^{-1/2} \), where the angled brackets show the cross-sectional average of \( M_0 \), and \( x \) was the longitudinal position. In turn, \( M_0 \) was giving by

\[
M_0 = (f^2 Q_0^2 / P_0) - P_0, \quad (5)
\]

where \( P_0 \) and \( Q_0 \) are complex functions of \( (h, f, \delta) \). Depth was represented by \( h \) and only depended on the \( yy \)-axis (\( yy \) was the transverse-channel direction at all the zones). The \( P_0 \) and \( Q_0 \) equations were trigonometric complex functions of three terms; in both cases the first term was independent of \( \delta \) (Winant, 2007). Him showed that when \( \delta \to 0 \), \( M_0 \) tended to \( h \) and the real part of \( \mu = \langle h \rangle^{-1/2} \) was independent of \( f \). If \( \delta \to \infty \), \( M_0 \) tended to \( -P_0 \) and also it was independent of \( f \).

Thus Winant (2007) found that the low order solution (equation 4) was practically not affected by rotation, which implies that the \( f \) term in the momentum equation is
negligible. Further information on calculating $P_0$, $Q_0$ and the $N_{\infty}N^{(0)}$ lower order solution can be found in Winant (2007).

4.2. Analytical solution

For this analysis $h$ was represented by the function

$$h = 0.01 + 0.99(1 - y^4).$$  (6)

The bottom shape was selected to represent dredged navigational channels (especially Zones 1 and 3). However, following Winant (2007) and Waterhouse et al. (2011), other theoretical bathymetric shapes were tested and produced similar results. Hence, the $N_{\infty}N^{(0)}$ solution depended on two non-dimensional parameters: $\kappa$ and $\delta$. The methodology of Henrie and Valle-Levinson (2014) was applied to obtain the best $N_{\infty}N^{(0)}$ solution using equation (4) for the system by solving for $\delta$ at each Zone with $K_1$ or $M_2$ tide frequencies. This method considers the range of eddy viscosity values $K$ and then calculates the range of $\delta$ parameters. The $\kappa$ parameter was known and constant in each Zone because it depends on the geometric and tidal characteristics.

In the study area, the geometric characteristics of the channels were met with model requirements. The $\kappa$ parameters can be calculated using geometric dimensions at each channel and the frequency of the tide ($K_1$ or $M_2$) tide frequencies. However, $\delta$ is undefined because the vertical eddy diffusivity is usually unknown. This method considers the wide range of $\delta$, from frictionless...
Therefore, the approach used by Henrie and Valle-Levinson (2014) was calculated and constant in followed to obtain the best \( N^{(0)} \) solution using \( (4) \) for the system by solving for \( \delta \) at each Zone because it depends on with \( K_1 \) or \( M_2 \) tide frequencies. This method was used, considering the geometric range of eddy viscosity values to find a range of \( \delta \) parameters, and tidal characteristics. However, this method requires also the range of \( \kappa \) with the same length of \( \delta \). \( \kappa \) range should included the value of \( \kappa \) calculated for each Zone. Hence, a range of combinations of \( \kappa \) and \( \delta \) parameters for each tidal frequency. Each \( \kappa \) and \( \delta \) combination yielded a \( N^{(0)} \) solution.

Around 4000 values of \( N^{(0)} \) were calculated with equation \( (4) \). The number of solutions depends on how large the range of \( \Delta K_1 \) is (the length of the \( K_1 \) range \( \delta \) was arbitrarily selected, but in all the cases was from \( \delta < 1 \) to \( \delta \gg 1 \)). Each \( N^{(0)} \) solution was compared to the amplitude of the principal harmonic of the observed tides at each station using the root mean square error. Therefore, the parametric space among spaces between \( \delta \) (axis \( x \)), \( \kappa \) (axis \( y \)) and minimum root mean square error (axis \( z \)) were generated for all stations and for diurnal and semidiurnal signals. Each parametric space contains a minimum root mean square error band. The minimum root mean square error bands indicate the best solutions; the specific \( \delta \) values were selected using the intersection with \( \kappa \) values at each station. The minimum root mean square error solutions produced optimal values of \( \kappa \) and \( \delta \) for each selected frequency and zones. Hence \( \delta \) values do not require the direct solution of the vertical eddy
diffusivity. Nonetheless, from δ-optimal values, the eddy viscosity can be calculated to each zone and frequency.

When frictional characteristics of the bodies of water are unknown, using this methodology it is possible to determine if the systems are highly frictional basins (δ > 1), slightly frictional (δ ≈ 1) or frictionless (δ < 1). The last two having equation (4) as the solution (Winant, 2007). But in the highly frictional case (δ > 1), the dynamical balance describing the longitudinal propagation of the tidal signal is reduced to pressure gradient and frictional forces—friction. For this situation the results of Henrie and Valle-Levinson’s (2014) methodology (optimal κ and δ solutions from equation (4)) was an approach only used for diagnostics and classification. Knowing that this study site is highly frictional, and following Winant (2007), equation (4) was not used because the wave is damped towards the closed end and instead, the equation for tidal longitudinal propagation (\( N^{(0)} \)) in highly frictional water bodies was taken as

\[
N^{(0)} = \exp[-(1 + i)\kappa\delta y x] \exp[-(1 + i)\kappa\delta y x],
\]

where \( N^{(0)} \) is a complex function defined by \( \delta, \kappa, x, \) and \( y \). The \( y \) parameter depends on the geometry of the basin and is of order 1 (Winant, 2007). To select the optimal values of \( y \) at each Zone, sensitivity analysis between 0.2 and 1.2 were performed. The best fit between observations and the model results gave the optimum values of \( y \). For Zone 1 (short channel) lower values of \( y \) (≈ 0.2) were used and for Zones 2 and 3 (long channels) midpoints values (≈ 0.7) were used. The \( N^{(0)} \) solution satisfies the 1-D diffusion equation, which is typical of...
highly frictional basins (LeBlond, 1978; Friedrichs, 2010). Hence, the solutions along each one of the channels were expressed as

\[ N_1^{(0)} = \exp[-(1 + i)\kappa_1 \delta_1 x_1], \quad (8) \]
\[ N_2^{(0)} = \exp[-(1 + i)\kappa_2 \delta_2 x_2][- (1 + i)\kappa_2 \delta_2 x_2], \quad (9) \]
\[ N_3^{(0)} = \exp[-(1 + i)\kappa_3 \delta_3 x_3][- (1 + i)\kappa_3 \delta_3 x_3], \quad (10) \]

where 1, 2, 3 stand for the zones.

### 4.3. Channel connections

The Lighthill (1978)'s approach was used to obtain the spatial variations of the tidal signal along the entirety of each branch, without losing the \( \delta \) and \( \kappa \) characteristics of each channel. The \( N^{(0)} \) solution is the non-dimensional attenuation ratio

\[ N^{(0)} = \frac{\eta^*}{C^*}, \quad (11) \]

for \( N_1^{(0)} \) at Zone 1, \( C^* \) was the observed tidal harmonic amplitude (diurnal and semidiurnal in this case), therefore \( \eta_1^* = (C^*)N_1^{(0)} \). The bifurcation area was located at \( x = l \), where \( l \) was the distance of the mouth to the bifurcation. The amplitude calculated at \( x = l \) of Zone 1 was used at the beginning of Zones 2 and 3 in the middle of the lagoon (bifurcation area, Fig. 1c) with \( N_1^{(0)}(x = l) \) as a common factor of in both channels. Therefore, the solution along each channel was expressed as

\[ N_1^{(0)} = \frac{\eta_1^*}{C^*}, \quad (12) \]
\[ N_2^{(0)} = \frac{\eta_2^*}{\eta_1^*(x = l)}, \quad (13) \]
The model was forced at the mouth by the observed diurnal and semidiurnal tidal amplitudes. The solution curves of (12), (13) and (14) were normalized with the total length of each branch, such that \( x^*/L_{W,E}^* \), where \( L_{W}^* \) and \( L_{E}^* \) are the total length of west and east branch, respectively.

When the Coriolis term was negligible, the lower order longitudinal local velocity solution \( U \) was function of \( N^{(0)}_2, p_w \) and \( k \), where \( p_w \) was complex function of \( k, x \) and \( \delta \). The longitudinal local velocity was depth-integrated to obtain horizontal transport in the longitudinal direction and for specific Zone was given by

\[
[U]_j = \frac{i \frac{\partial N^{(0)}_j}{\partial x}}{\kappa},
\]

where the \( j \) subscript indicates the solution Zone (1, 2 and 3) and is directly related to the solutions along each channel (1, 2 and 3, in equations (12), (13) and (14)), respectively). The longitudinal transport \( [U]_j \) was calculated for each channel using in each \( \frac{\partial N^{(0)}_1}{\partial x}, \frac{\partial N^{(0)}_2}{\partial x} \) or \( \frac{\partial N^{(0)}_3}{\partial x} \): at the middle of the lagoon (closed end of Zone 1), \( [U]_j \) satisfied the continuity condition (Lighthill, 1978).

5. Results
5.1. Sea surface elevation

The quality of the time series obtained from the pressure sensors (Fig. 2) inside the lagoon was verified by statistical comparison between the time series of the moored current profiler (used as a reference instrument) at the mouth (Fig. 5a) and the Diver CTD pressure signals throughout the lagoon. Because the Diver CTD sensors were operating correctly, the correlation between the two time series was close to 1. The sea surface elevation time series showed an attenuation and distortion of the tidal signal toward the heads (Fig. 2). The signals at C1 and C3 presented minimal distortion with respect to the inlet, with well-defined diurnal variations. However, semidiurnal variations during neap tides were distorted as broad and hooked waves. The C2 station was the shallowest and presented enhanced distortion at low water, especially in spring tide because during the lowest water levels the sensor was out of the water. The distortion observed in stations C1, C2 and C3 could be explained by friction effects in shallow water (LeBlond, 1978). In addition to the diurnal and semidiurnal oscillations observed in the time series, longer period fluctuations are visible. Atmospheric pulses were present in sea surface elevation variations as observed with the southerly Hurricane Ernesto winds, which caused a negative anomaly from 7 to 10 August 2012 (Fig. 2).

The most energetic tidal frequencies observed in the sea surface elevation were distinguished in the power spectra of the six time series (Fig. 3). The spectra showed three characteristic peaks: the diurnal band (~0.04 cph) was the most
energetic, the low-frequency band (0.0042 to 0.0023 cph), and the semidiurnal band (~0.08 cph). Remarkably, the semidiurnal band was not statistically significant on the western side of the lagoon, unlike the rest. The time series of the west head was compared to that at C1 with the frequency response function. The gain diagram and phase diagram were obtained from the cross-spectra of sea surface elevation between station C1 and the west head (Fig. 4), and indicated that the causeway acted as a low-pass filter. Coherence was also calculated (not shown), the low frequency band presented coherence values of around 0.6 and practically zero phase lags (Fig. 4, lower row), the rest of the bands showed lower coherence values than 0.6, therefore these bands were not significant. The response function analyses provided a quantitative explanation of the drastic attenuation and phase of the semidiurnal signal beyond the causeway. Another significant feature was the contribution of the third-diurnal band (~0.12 cph), especially in the middle of the lagoon. Furthermore, a low-frequency peak (~0.0027 cph equivalent to a period of 15 days) appeared in the entire lagoon. Mechanisms that could change the sea surface elevation with a periodicity of approximately 15 days were narrowed down to the wind (not discussed here) and a forced fortnightly tide in a frictional basin.

Kantha (2005) stated that the fortnightly variation in the Yucatan Shelf is modulated by the $K_I$ and $O_I$ interaction. Following Kinsman (1984), the modulation period ($T_{mod} = 4\pi/\Delta\sigma$, where $\Delta\sigma$ is the difference between $K_I$ and $O_I$ frequencies) was 13.7 days, and the modulation amplitude was 0.3 m. However, in the case of $M_2$ and $S_2$ the modulated period was 14.3 days and the modulation amplitude was
The frequency spectra were utilized to estimate the low-frequency amplitude at each station (Fig. 3). At zones 1 and 3 the low-frequency amplitudes were slightly amplified (from ~0.7 m at the mouth to ~1.1 m at east head), meanwhile at Zone 2 the signal was attenuated (~0.45 m) by the causeway effect. Even considering this attenuation, the differences between the observed and predicted low-frequency modulations were important at all sites. This suggested that low frequency sea level modulation might not only be caused by astronomic influence.

5.2. Currents

A harmonic analysis was performed to the along-channel velocities observed at the mouth (Fig. 5b) to determine amplitude, phase, and percentage of explained variance for the tidal components (Pawlowicz et al., 2002). Power spectra were calculated for the vertically averaged longitudinal velocity (Fig. 5c) and for each depth of the longitudinal velocity (not shown). The diurnal band was the most energetic throughout the water column, and maximum at the surface. The second most important energy peak appeared in the semidiurnal band, followed by third-diurnal energy that illustrated non-linear effects on the tidal flows (Fig. 5c).

The main tidal harmonics of the currents were explored with a tidal ellipse analysis (Table 1). These results were obtained with the vertically averaged velocity. The
diurnal $K_1$-current and $O_1$-current showed amplitudes of $23.2 \pm 1.9$ and $25.3 \pm 2.4$ cm/s, respectively, with the $O_1$-current exhibiting less snr than the $K_1$-current. The semidiurnal principal components were $M_2$-current and $N_2$-current ($12.1 \pm 0.8$ and $3.5 \pm 0.8$ cm/s, respectively); with the lowest snr observed for the $N_2$-current. The other semidiurnal constituent, $S_2$, was considered insignificant inside the lagoon because its amplitude was the smallest at the mouth ($2.0 \pm 0.9$ cm/s). The semiminor axis was small compared with the semi-major axis (ellipticity was $\sim 0.05$), which indicated quasi-rectilinear tidal flows (elongated ellipses); it was due to the narrow Chelem mouth. The diurnal ellipses rotated counterclockwise because their semi-minor axis was positive. The $M_2$ rotated clockwise because it showed a small negative semi-minor axis; however this result was not reliable because its error associated error to the semi-minor axis was of similar magnitude and the measured amplitude of as the axis was of the same order of magnitude to the instrument's accuracy ($\pm 0.5$ cm/s). semi-minor axis value. The orientations of all the ellipses were consistent with the lagoon entrance, close to $90^\circ$ counterclockwise from the east.

Tidal current ellipses were also calculated for each bin depth to obtain the vertical structure (Fig. 6). The magnitudes of the semi-major and semi-minor axes for the main diurnal components ($O_1$ and $K_1$) were similar when considering the error, but twice as large as that of the semidiurnal $M_2$ (Table 1). The vertical structures of the magnitude and the phase for the semi-minor axes were practically homogeneous throughout the water column (Fig. 6 b, d). However, the vertical structures of the semi-major axes (Fig. 6a) in the tidal current components showed parabolic
behavior (decreasing towards the bottom), because explained by the influence of bottom friction (MacCready and Geyer, 2010).

5.3 . Tidal spatial variations

The harmonic analysis of the sea surface elevation yielded the main components for the diurnal and semidiurnal frequencies and only one significant compound tide (Table 2). The main diurnal components were $K_1$ and $O_1$ with amplitudes of 17.8 ± 1.4 and 17.3 ± 1.8 cm, respectively. The main semidiurnal components were $M_2$ and $N_2$ with amplitudes of 5.7 ± 0.3 and 2.0 ± 0.2 cm, respectively. The semidiurnal component $S_2$ had an amplitude of 1.1 ± 0.3 cm at the mouth, but was found to be insignificant toward the west and the east ends.

The system appeared to be dominated by diurnal tides. The type of tide was characterized by the form number ($F$), defined as $F = (K_1 + O_1) / (M_2 + S_2)$ (Defant, 1958). The form number was between 5 and 8 for all sites, which indicates that the entire lagoon is diurnal dominant. The highest $F$ value was found in the west head where the differences between diurnal and semidiurnal components were greatest. The tidal range was approximately 0.7 m at the mouth and decreased toward the heads at all sites except at the west head. On the other hand, the fortnightly declinational variation $|(K_1 + O_1)/(K_1 - O_1)|$ was larger than the fortnightly synodical variation $|(M_2 + S_2)/(M_2 - S_2)|$. At Zone 1, the fortnightly declinational variations were two orders of magnitude larger than the fortnightly
synodical variation. This difference was more pronounced toward the lagoon heads where the amplitudes became minimal. The maximum difference between these fortnightly variations was found at the mouth where the difference between the two amplitudes was largest.

None of the overtides was significant. The $MK_3$ compound tide was generated by the non-linear $K_1-M_2$ interactions and was observed in the entire lagoon, in particular at Zone 3, where it reached amplitude of $1.4 \pm 0.7$ cm (Table 2). The attenuation of $MK_3$ amplitude at Zone 2 was caused by the strong semidiurnal signal attenuation and small $K_1-M_2$ interaction. In systems dominated by diurnal tides, shallow water tidal components are normally generated in the third-diurnal band ($\sim 0.125$ cph) (Dworak and Gomez-Valdes, 2005). According to the analytical solutions for the interactions between two-tide components ($K_1$ and $M_2$) proposed by Parker (1991), the quadratic part of the nonlinear friction term, $u \frac{\partial u}{\partial x} + \frac{\eta u}{h} |u|/h$, accounts for the shallow water tidal components. A scaling analysis was performed between the three nonlinear terms (the advective $u \frac{\partial u}{\partial x}$ and the frictional terms $\eta u |u|/h^2$ and $u |u|/h$) to demonstrate their relative importance. The magnitude of the tidal velocities ($u$) was of the same order $10^{-1}$ with a longitudinal scale, $x$, of order $1 \times 10^3$ and the depth ($h$) of order 1. Therefore the magnitude of $1/h |u|/h$ was at least two orders of magnitude larger than the two other terms of the one-dimensional momentum equation ($1/h |u|/h \gg u \frac{\partial u}{\partial x} + \frac{\eta u}{h} |u|/h \gg u \frac{\partial u}{\partial x}$).
Furthermore, since the study area is a highly frictional lagoon, the balance between the pressure gradient and frictional forces controls it. Following Hill (1994), the balance was written as $\frac{\eta_s - \eta_l}{L} = C_{friction} \left( \frac{\eta_s - \eta_l}{\eta_m} \right)$ where the sea surface elevation in the open end was $\eta_s$, the sea surface elevation in the lagoon was $\eta_l$, $\eta_m$ was the mean surface elevation between them, $C_{friction}$ was the dimensionless friction coefficient. Consequently the quadratic part of the non-linear friction term $\frac{1}{2} \| u \|^{2}$ was expected as shallow water tidal generator.

The non-linear distortion between the $M_2$ and $M_4$ components is frequently used to determine the dominance between flood and ebb (Friedrichs and Aubrey, 1988). However, Ranasinghe and Pattiaratchi (2000) showed that this criterion does not apply to lagoons with diurnal dominance. Since the studied area is considered a shallow system, the method proposed by Friedrichs and Aubrey (1988) was applied. The ebb-flood asymmetry can be understood by analyzing the behavior of the tidal phase speed as a function of changes in width and depth of the basin. When a deep basin system is dominated by high tide propagated faster and partially catches up with the previous low tide. This situation produces a shorter rising tide and flood dominance. In the case of a wide basin system dominated, low tide propagated faster partially catching up with the previous high tide, therefore, produces a shorter falling tide and ebb dominance. Friedrichs and Aubrey (1988) proposed ebb and flood-dominant areas at the parametric space between $(a/\langle h \rangle)$ and $(V_s/V_c)$. The first parameter is the ratio between tidal amplitude and the average channel depth. The second
parameter is the ratio between the volume of storage in intertidal zones $V_s$ and the volume of the channels $V_c$. In Chelem, these relations are small in all three zones ($a/(\langle h \rangle \sim 0.1)$ and $(V_s/V_c \sim 0.1)$, because the average depth of all zones is small but larger than the tidal amplitude. The combination of the two ratios classifies the system as ebb-dominant.

The harmonic analysis was also used to estimate tidal attenuation and phase of the sea surface elevation (Table 2). Diurnal signal attenuations were greatest at the end of Zones 2 and 3, and the time delay observed were of $\sim8$ h and $\sim3$ h at the west and east heads, respectively. The maximum semidiurnal attenuation was $\sim5$ and $\sim4$ cm with a time delay of $\sim5$ and $\sim3$ h at the end of Zones 2 and 3, respectively (Table 3). In Zone 2, the semidiurnal signal was practically filtered out. Our results indicated that frictional effects were the main attenuators of the tides. This was corroborated by the application of an analytical model that required frictional effects to resemble observations, as explained next.

5.4. Model results

Following Henrie and Valle-Levinson (2014), Winant (2007)'s model was applied piecewise to Chelem. This required calculation of the $\delta$ and $\kappa$ parameters for each zone (Fig. 7a; Table 3). Values of $\delta > 1$ throughout the lagoon indicated that the lagoon was highly frictional; therefore, (7) was the solution used. Zones 1 and 3 exhibited the greatest $\delta$ and smallest $\kappa$ values. Zone 1 also showed the least
amplitude attenuation. Zone 2 showed the lowest $\delta$ and the highest $\kappa$ values because of its dimensions. From optimal values, the eddy viscosities were calculated to each zone and frequency. The eddy viscosity was approximately the same for diurnal and semi-diurnal frequency, but it was varied at each zone. The vertical eddy viscosities were approximately $0.02 \text{ m}^2/\text{s}$ at Zones 1 and 3, this value was similar to that found by Munk (1966). At the Zone 2 the vertical eddy viscosities were approximately $0.003 \text{ m}^2/\text{s}$, coinciding with the values reported by Waterhouse et al. (2011) for these type of channels.

Once the $\delta$ and $\kappa$ parameters were determined for each zone and for each tidal harmonic of interest. The Figure 7a only shows the $\delta$ and $\kappa$ parametric space for the diurnal frequency in Zone 2 as an example, but the same analysis was performed for each zone and frequency. In this figure, the optimal $\delta$ value at the intersection of minimum root mean square error band and $\kappa$ value of this station was represented with the white line (Fig. 7a). The model of Winant (2007) effectively described the tidal wave attenuation pattern with respect to observations at each station (Fig. 7b). The x-axis in Fig. 7b and 7c was normalized by the total length of each branch. Tidal signal attenuation was observed at all stations inside the lagoon and the attenuation magnitude increased toward the west and east heads (Table 3). Tidal phases were calculated with the model and the phase differences with respect to the principal tidal components of the inlet were listed on Table 3. The analytical time delay diverged from the time delay observed (around 3.2 h) at the end of Zone 2 (west head) especially with the
diurnal signal, while those of Zone 1 (0.1 h) and Zone 3 (0.6 h) were consistent (Fig. 7c). The analytical model describes the linear problem, the study area was highly frictional and the balance between the pressure gradient and frictional forces controlled it. The fit between observations and the model results was suitable except in the causeway zone where advection terms may become relevant because of the narrow-channel effects. The model limitations were more noticeable at Zone 2 especially in the phase differences between observations and model results. The phase differences were because the model did not consider the causeway’s effect directly.

Once the spatial variations of sea surface elevation were determined, the next step was to calculate the spatial variations of the longitudinal transport and phase within each zone for the semidiurnal (Fig. 8) and diurnal signals (Fig. 9). To verify the model results, the tidal transport observations at the inlet were compared to model results at the beginning of Zone 1, using an approximate cross sectional area of 450 m². Analytical results were consistent with the observations. The observed diurnal (semidiurnal) tidal transport was ~110 m³/s (~50 m³/s) and the maximum value of the analytical result was ~100 m³/s (~40 m³/s). Analytical results showed that ~35% of the diurnal longitudinal transport goes to Zone 2, and ~65% to Zone 3. Similar results were obtained for the semidiurnal longitudinal transport (~30% to Zone 2 and ~70% to Zone 3). These differences in each zone were because δ and κ differ in each zone, hence the propagation characteristics generate different longitudinal transport at each channel. The semidiurnal signal in the longitudinal transport was the most attenuated toward the heads: ~0.9 and ~5
m³/s at Zones 2 and 3, respectively. Time delay between the east head and both the end of Zone 1 and the mouth was ~2.5 and ~2.7 h, respectively (Fig. 8). The diurnal signal was also attenuated but it reached the closed ends of Zones 2 and 3 as ~10 and ~20 m³/s, respectively. The time delay in the diurnal transport toward Zones 2 and 3 was 4.3 and 3.5 h, respectively (Fig. 9).

6. Discussion

The tidal signal in the Yucatan Shelf is mainly diurnal as previously described by Kantha (2005) and Kjerfve (1981). Present results showed that the tidal signal in Chelem is also diurnal but with a slight semidiurnal influence. The principal diurnal components were $K_1$ and $O_1$ and the principal semidiurnal components were $M_2$ and $N_2$. The semidiurnal component $S_2$ was small in the lagoon because $M_2$ and $S_2$ amphidromic points are located in the middle of the Yucatan Shelf, and the region for $S_2$ minimum amplitude is larger than for the $M_2$ (Kantha, 2005). The tidal-ellipses from the lagoon entrance were consistent with the harmonic analysis results using the sea surface elevation data and with previous reports of tides in the GoM. David and Kjerfve (1998) reported that at the inlets of the Terminos lagoon (located ~400 km southwest of Chelem) the $K_1$-current and $O_1$-current were the main diurnal components and the $M_2$-current was the main semidiurnal component. The magnitudes reported in these cases were similar to those obtained in Chelem.
At the Chelem inlet, the phase difference between $K_1$ water level and the $K_1$ current was $\sim 49^\circ$ (Table 1 and 2). This behavior is typical of highly frictional regions and it is consistent at the Chelem inlet. In this situation, theoretically the current and sea surface elevation phase difference should be $45^\circ$ (Winant, 2007). Friedrichs (2010) showed that in long, shallow, non-convergent estuaries friction dominates acceleration in the momentum balance. Therefore, the 1-D dynamics has the form of a 1-D diffusion equation.

Although this work did not intend to classify the lagoon morphologically, the behavior of the west channel (beyond the causeway) corresponded to a choked system as described by Kjerfve and Magill (1989). The tidal signal in Chelem reached the western branch as it passed through two gates in the causeway. In this process, frequencies higher than the diurnal signal were filtered out. Kjerfve and Magill (1989) quantified the filtering of $K_1$ and $M_2$ harmonics in choked systems using the coefficient of repletion ($R$). This coefficient provided a quantitative measure of filtering characteristics at the inlet channels and was calculated as

$$R = \left( \frac{T}{2\pi a_0} \right) \left( \frac{A_C}{A_B} \right) \left[ \frac{2g a_0}{(1 + 2g L n^2 r^{-4/3})} \right]^{1/2},$$

(16)

where the tidal period is $T$, the tidal range in the adjacent body of water is $2a_0$, $A_C/A_B$ is the relation between the cross-sectional channel area and the surface basin area, the gravity is $g$, the length of the entrance is $L$, the hydraulic radius of the channel is $r$, and $n$ is the Manning’s channel friction range (between 0.01 and 0.10 s/m$^{-1/3}$). For the analysis of the Chelem causeway effect, the tidal periods ($T$)
were $K_1$ and $M_2$, the tidal range at the Zone 1 (adjacent body of water for this case) was 0.8 m, the relation between the cross-sectional channel area and the surface basin area at Zone 2 was $A_c/A_B \sim 5.9 \times 10^{-6}$, the gravity is $g$ (9.8 m/s$^2$), the length of the entrance was $L \sim 20$ m, the hydraulic radius of the channel was $r \approx 1.75$ m (rectangular shape channel was considered), and Manning’s friction range at the channel was taken as 0.10 s/m$^{1/3}$. The non-dimensional coefficients of repletion were calculated for both $K_1$ and $M_2$ harmonics as 0.24 and 0.12, respectively, corresponding to high choking regimes (Hill, 1994). A frequency response function analysis between the C1 signal and the west head signal confirmed such filtering. The semidiurnal amplitude of 0.5 cm obtained from the harmonic analysis was of the same magnitude as the resolution of the instruments and therefore could not be properly measured.

Geometric parameters determine spatial variations of coastal tides (Friedrichs, 2010). The relative importance of the geometric characteristics was represented by the $\kappa$ parameter, which was $>1$ throughout the lagoon. In fact, the diurnal tidal wavelength was two orders of magnitude longer than the basin length ($\sim 11.5$ km between the inlet and the end of Zone 2), and more so with the shorter basins at Zones 1 and 3. Each zone had different $\kappa$ values, related to their specific geometric characteristics. Zones 1 and 3 had similar $\kappa$ values (0.05 and 0.08, respectively) for diurnal and (0.10 and 0.15, respectively) for semidiurnal signals. Both the diurnal and semidiurnal wavelengths were longer than the combined length of Zones 1, 2 and 3.
According to Friedrichs (2010), a tidal signal becomes altered by deformation and attenuation because of frictional effects when it enters a water body. Both phenomena (deformation and attenuation) are observed from the mouth towards the heads (Fig. 2). In highly frictional lagoons, local acceleration and Coriolis forces are considered insignificant (Winant, 2007). Consequently, the balance between pressure gradient and friction forces determines tidal propagation. Our results showed that Chelem is a highly frictional lagoon because the frictional parameter $\delta > 1$ for both the diurnal and semidiurnal signals in all zones. Frictionally, Zones 1 and 3 had close $\delta$ values for both diurnal (15.7 and 14.7 respectively) and semidiurnal signals (8.4 and 9.2 respectively). The diurnal and semidiurnal frictional parameters at Zone 2 were 9.4 and 7.3, respectively.

Geometric characteristics at Zone 2 produced the highest values of the geometrical parameter; these high values increased the relative importance of the pressure gradient in the highly frictional momentum balance. This compensation (pressure gradient dominated) between the pressure gradient and geometric characteristics is greater at Zone 2 (especially in the semidiurnal case) and the frictional parameter at Zone 2 was the lowest.

Consequently, the results of the sea surface elevation analysis and longitudinal transport demonstrated that Zone 2 had incurred the greatest attenuations because of a physical restriction (causeway), even though it displayed the least friction. Theoretically the model was not sensitive to the causeway, with the solution based
on the geometric and frictional parameters (Winant, 2007). However, frictional parameters were optimized by reducing the root mean square error between model results and observations (Henrie and Valle-Levinson, 2014). Therefore, the estimated δ values included the frictional effects evident in the observations that could involve for example, physical restrictions and changes in bottom roughness.

In addition to the observed tidal frequencies, low frequency variability (~15-day period) was present in the entire lagoon. These low frequency signals could be generated by the non-lineal interactions of the major tidal harmonics (beats or modulated frequency). However, fortnightly variations in the tides can be influenced by bottom friction, as has been regularly observed in highly frictional semi-enclosed basins with narrow and shallow inlets connected to the ocean. This effect is caused by increased friction and pressure gradients that produce higher lagoon water levels. The intensity of this phenomenon depends on the spring and neap tide cycles, where spring tides would intensify the effect while neap tides would minimize it (LeBlond, 1979; Hill, 1994). Since the Chelem fortnightly declinational variations were larger than the fortnightly synodical variations, the most likely synchronization of the spring-neap cycles is with the 27.32 days orbital cycle of the Moon (tropical month). Another factor that contributes to low frequency oscillations is the wind. There is a well-supported hypothesis about the influence of the E-W wind stress on subtidal variations in lagoons (Gutiérrez de Velasco and Winant, 2004), but this effect has not been studied in Chelem. The effect of winds on subtidal variations should be examined in a future study.
7. Conclusions

This study showed that tidal currents inside a shallow semi-enclosed branched system were driven by the barotropic pressure gradient force acting over a short distance equivalent to ~1/10 the tidal wavelength. The effects of a causeway were accounted for as frictional and geometric parameters were optimized according to observations. Chelem was determined to be a highly frictional coastal body of water where pressure gradient and frictional forces control the hydrodynamics. The tidal signal was strongly attenuated as it propagated within the lagoon because the geometric characteristics of the lagoon contributed to the high frictional effects. The tidal signal was most attenuated toward the west head as the causeway acted as a hydraulic low-pass filter. This illustrated the effects of anthropogenic modifications to coastal lagoon systems, which clearly alter tidal dynamics.

Acknowledgments

We are grateful to Gilberto Jeronimo Moreno for providing the data and to CONACYT’s M0023- FOMIX YUCATAN project for supporting this study. We also thank CONACYT for granting scholarship No. 206018 to Leonardo Tenorio-Fernandez. AVL acknowledges support of NSF project OCE-1332718. We also acknowledge the critical comments from the reviewers.
References


Figure captions

Figure 1. - a) Location of Chelem in the Yucatan Peninsula, Mexico. b) Chelem sampling locations with the black dots representing the locations of moored Diver CTD, a black ring represented the location of the current profiler (Aquadopp) and Diver CTD moored at the mouth, gray contours show approximate depth. At the left hand corner is represented the channel network. c) Analytical model setup, showing Zones 1, 2 and 3, with the x axis representing the along-channel direction within all the zones.

Figure 2. - Time series of the sea surface elevations in cm for all measurement stations spanning from 27 June 2012 to 25 August 2012.

Figure 3. - Power spectra estimations normalized by the mean for all measurement stations using the sea level elevation data obtained from a 60-day time series the (black line and crosses). The gray band represents the confidence interval of each set of data and the black continuous line the 95% confidence level.
Figure 4. - Response function between the cross-spectral density of sea surface elevation at station C1 and the west head. Upper row: gain diagram, the vertical lines are the error bar of gain values. Lower row: phase diagram. In both diagrams, the crosses indicate the intersection between the frequency and the gain or phase values.

Figure 5. - (a) Sea surface elevation measured with the Aquadopp-sensor of pressure spanning from 27 June 2012 to 25 August 2012. (b) Longitudinal velocity at each depth yielded at the inlet by current profiler measurements, same time span. Positive velocities represent inflow and negative velocities represent outflow. (c) Power spectrum (PSD) in (m/s)^2/cph of the depth-averaged longitudinal velocity.

Figure 6. - Vertical structure of the ellipse tidal current parameters: the amplitude of the a) semi-major and b) semi-minor axis (cm/s), c) orientation in degrees of the semi-major axis relative to the East, and d) the phase in degrees of the tidal potential relative to Greenwich.

Figure 7. - (a) Example of optimal $\kappa$ and $\delta$ values obtained by root mean square error analysis of observational data and analytical solutions for the diurnal frequency in Zone 2. (b) Comparison of the tidal amplitude attenuation of observational data with analytical solutions ($N^0$) of diurnal and semidiurnal tides in all three zones of Chelem, using the solution for $\delta>1$. (c) The corresponding phase
difference for the tidal amplitude attenuations of (b). In both (b) and (c), the $xx$-axis are normalized with the total length $L$ of the west and east branch (beginning at the mouth). The western and the eastern channels are represented with blue and blacks lines, respectively. The solid lines are the diurnal signal and the dashed lines are the semidiurnal signal. At the beginning and at the ending of each zone the observations are represented by circles for the semidiurnal signal, and by crosses for the diurnal signal.

Figure 8. - Longitudinal transport $[\mu_2]$ of the semidiurnal tide ($M_2$) in m$^3$/s. White lines represent the phase for a) Zone 3 (eastern side), b) Zone 1 (central part), and c) Zone 2 (western side).

Figure 9. - The same as Fig. 8, for the diurnal tide ($K_1$).

Legends to tables

Table 1: Top section: geographic location of the measurement stations and average dimensions of each zone: length ($l$), width ($W$), and depth ($H$). Lower section: ellipse parameters of the most important tidal currents for the vertically averaged velocity at the inlet of Chelem. $M$ is the amplitude of the semi-major axis in cm/s, $m$ is the amplitude of the semi-minor one in cm/s, $\phi$ is the inclination in
degrees, and \( \theta \) is the phase in degrees. The error of each parameter is denoted by \( \delta M, \delta m, \delta \phi, \) and \( \delta \theta \); and \( snr \) indicates the signal-to-noise ratio.

Table 2: Principal tidal constituents and compound tide extracted from sea surface elevation data obtained in a period of 60 days from Chelem measurement stations: Mouth (M), C1, West Head (WH), C2, C3, and East Head (EH). Amplitude (A) in cm, phase (\( \theta \)) in degrees, amplitude error (\( \delta A \)) in cm and phase error (\( \delta \theta \)) in degrees. Values less than 1 cm were excluded.

Table 3: Observations and analytical model results of the sea surface elevation attenuation and time delay of diurnal and semidiurnal signals at the three zones. The form numbers (\( FE \)) were estimated using the observations; \( \kappa \) and \( \delta \) are the optimal values for each zone; \( \eta_c/\eta_i \) is the attenuation fraction relative to the inlet for both signals at each zone; \( \theta_c - \theta_i \) is the signal time delay in hours relative to the inlet. \( \eta_i \) and \( \theta_i \) are the sea surface elevation amplitude and phase at the mouth, and \( \eta_c \) and \( \theta_c \) are amplitude and phase at the closed end of each zone.
Tidal dynamics in a frictionally dominated tropical lagoon

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Re-submitted to

Continental Shelf Research

October 2015
Abstract

This study examined the dynamics of tidal propagation inside a tropical lagoon. Sea surface elevation (inside) and current profiles (at the inlet) were examined over 60 days at the Chelem lagoon, which is a branched tropical lagoon located in the northern Yucatan Peninsula. Tides were predominantly diurnal with a wavelength at least 20 times longer than the total length of the basin. Spatial variations of sea surface elevation and the longitudinal transport were described in each branch by applying a linear analytical model and the results were compared to observations. Results showed that the coastal lagoon was highly frictional. The tidal signal was attenuated between 30% and 40% toward the lagoon heads, a result of the balance between pressure gradient and frictional forces. A causeway that choke the western side of the lagoon allowed the propagation of the diurnal signal toward the west head of the basin but damped the semidiurnal signal. The causeway acted as a hydraulic low-pass filter, as in natural choked systems. The causeway’s filter effect was included in the analytical model by optimizing the frictional parameters.

Keywords: Choked coastal lagoon; tidal hydrodynamics; highly frictional; Chelem lagoon.
1. Introduction

Coastal lagoon hydrodynamics are influenced mainly by tides, winds, heat fluxes, and freshwater inputs. The dynamics of coastal lagoons are also shaped by inlet morphology. Coastal lagoons may be subdivided according to their inlet characteristics into leaky, restricted and choked systems (Kjerfve and Magill, 1989). Leaky lagoons are longest and narrowest (~$10^3$ and ~$10^2$ m, respectively), parallel to the coastline and their hydrodynamics are controlled directly by the ocean through many inlets. Restricted systems are typically oriented parallel to the coastline, have one or two inlets and a well-defined tidal circulation, where the dynamics are controlled by the adjacent ocean. Choked lagoons are usually found along high wave energy coastlines with marked littoral drift and have one or more narrow inlets. Dominant wind forcing and freshwater pulses that may produce intermittent vertical stratification characterize these lagoons. In choked systems, the tidal signal is altered or eliminated because the inlet acts as a dynamic low-pass filter (Kjerfve and Magill, 1989). Choked systems, especially those located in the tropics, are a very important part of the local ecology owing to their high primary and secondary production (Krumbein et al., 1981; Barnes, 1980). Therefore, understanding their dynamics is essential because they play an important ecologic and economic role (Albrecht and Vennell, 2007).

In general, the hydrodynamics of choked lagoons are poorly understood. Such is the case in most of the lagoons that line the Mexican coast of the Gulf of Mexico (GoM). Considering this dearth of knowledge in the tropics, the approaches that
have been used in subtropical and temperate regions can also be applied to study these systems. The purpose of this investigation is to analyze the dynamics of tidal propagation inside a branched tropical lagoon with predominant diurnal tides. The tidal propagation dynamics of the lagoon were analyzed using a linear model proposed by Winant (2007). This study optimized different frictional and geometric parameters on the basis of observations in the lagoon, following the approach by Henrie and Valle-Levinson (2014). Analyses of the data and of the analytical model output provided the frictional characteristics of the two branches of the lagoon.

2. Study area

The Chelem lagoon is located between 21°10’ and 21°19’N and between 89°47’ and 89°37’ W (Fig. 1b). It is a branched tropical lagoon, which extends parallel to the coast in an E-W orientation, and is shallow with depths ranging from 0.7 m to 3.5 m. Since 1969, many physical modifications have affected Chelem. The most drastic alteration was the construction of a causeway across the lagoon in the 1980s. This causeway divided the lagoon and severely restricted water circulation toward the western head. The causeway has two bridges, each one with 5 m-wide gap that allows some water flow and maintains the west head connected to the central lagoon. However, this restriction modifies the natural hydrodynamics (e.g., see Hill, 1994) and has already caused diverse environmental problems.

The lagoon is a network of three zones connected by the central lagoon area (Fig. 1b). Zone 1 is a meridional channel that connects the lagoon with the Yucatan
Shelf. This Zone is the deepest, narrowest, and shortest (Table 1 shows the average dimensions of each zone). Zones 2 and 3 bifurcate from Zone 1 and are channels oriented westward and eastward, respectively. Zone 2 is the longest, shallowest, widest and is affected by the causeway. It is connected with Zone 1 only through the causeway gaps. Zone 3 extends from Zone 1 to the eastern head, which has medium length, medium width, and medium depth.

*Chelem* tidal signals related with those of the Yucatan Shelf that are forced by the GoM tides (Fig. 1a). Tides in the GoM are the result of indirect tidal oscillations from the Atlantic Ocean and direct astronomical forcing (Zetler and Jansen, 1972). The main tidal constituents within the GoM are the lunisolar diurnal ($K_1$), the lunar diurnal ($O_1$), and the principal lunar semidiurnal ($M_2$); however, tide behavior varies along the coast (Kantha, 2005). The tides range from semidiurnal near Florida to diurnal at the Yucatan Peninsula according to tidal station information from the National Oceanic and Atmospheric Administration. David and Kjerfve (1998) found that at the Terminos lagoon in the neighboring Campeche state, Mexico (~400 km from *Chelem*) the tidal range is 0.3 m with diurnal dominance; this pattern continues all the way to the Yucatan Shelf.

Kjerfve (1981) studied the main tidal amplitudes and tidal phases along the Yucatan Shelf. He found that the amplitudes of the diurnal components $K_1$ and $O_1$ at the Progreso harbor (located ~10 km east of *Chelem*) were 17.7 cm and 17.1 cm, respectively, with a $M_2$ as the largest semidiurnal amplitude component; with a value of 6.0 cm. Martínez-López and Pares-Sierra (1998) used a three-dimensional
model to study tidal dynamics in the Yucatan Shelf. However, the tides within
Chelem, or any lagoon in Yucatan, have yet to be described.

The tidal signal inside semi-closed coastal basins is modified by friction, the Earth’s
rotation and morphology (Waterhouse et al., 2011). Frictional effects dominate over
the Earth’s rotation in shallow basins (Winant, 2007). For highly frictional basins,
the relation between sea surface elevation and water velocity departs from the
classical wave equation solutions (frictionless). Therefore, tidal propagation in
these systems could be described using the diffusion equation (LeBlond, 1978;
Friedrichs, 2010). This diffusive process suggests that greater friction induces
faster tidal amplitude attenuation. Under these conditions, the equation describing
the tidal signal propagation is one-dimensional along the basin and is only
determined by the pressure gradient and frictional forces, regardless of lateral
depth variations along the channel (Li and Valle-Levinson, 1999; Waterhouse et
al., 2011). These premises are explored in Chelem with observations and
analytical model results.

3. Materials and methods

Data used in this study consisted of a series of moored instruments: a current
profiler at the mouth and six hydrographic instruments throughout the lagoon. At
the entrance to Chelem, a velocity time-series was obtained from a bottom-
mounted, upward pointing Aquadopp Doppler current profiler (Fig. 1b and Table 1).
The time series spanned from 27 June 2012 to 25 August 2012 (60 days). The
instrument was deployed at the center of the lagoon’s navigation channel where the average depth is 3.5 m with respect to mean sea level. Observations were divided into 10 cells, each of 0.3 m length, with a blanking distance of 0.5 m at the bottom and 0.3 m at the surface (i.e., the last cell); thus, the profiles ranged from 0.5 m to 3.20 m above the bottom. The velocity profiles were obtained every 60 s and saving one averaged value every 15 min. The accuracy of the current profiler is 1% of the measured value, typically ±0.5 cm/s.

To provide a more extensive description of the tides within the lagoon, water level data were obtained from six deployed Schlumberger Diver conductivity-temperature-depth recorders (CTDs). Sampling interval for all Diver CTDs was 10 minutes. The present study concentrated on tides so it only analyzed water level data. One of the Diver CTDs was deployed at the mouth next to the current profiler, and the other five were inside the lagoon (Fig. 1b). The accuracy of the pressure sensor was ±4.905 × 10^-4 bar (~0.5 cm depth).

At Zone 1, a current profiler and Diver CTD were deployed at the mouth and a Diver CTD at station C1 (Fig. 1b and Table 1). In Zone 2, one Diver CTD was affixed at the west head. In Zone 3, Diver CTDs were moored at C2, C3 (in the middle of the channel) and at the eastern head. High frequencies and noise were removed from sea surface elevation data with a Lanczos filter (cut-off frequency of 0.25 h^-1). Power spectrum was calculated for each time series, with a 95% confidence interval (Emery and Thomson, 2001).
The spectral relationship between station C1 signal and the west head signal was evaluated to understand the effects of the causeway on the west channel. The frequency response function \( R = \frac{S_{xy}}{S_{xx}} \) was used to analyze the impact of the causeway, where \( S_{xy} \) is the cross-spectra between station C1 (input signal) and the west head (output signal), and \( S_{xx} \) is the auto-spectrum of station C1 signal, the response function is defined as the spectral gain and the phase between two signals (the input and output). The confidence level of 95 % was calculated using the coherence function, which is defined as \( C^2 = \left| \frac{S_{xy}}{S_{xx}S_{yy}} \right|^2 \), where \( S_{yy} \) is the auto-spectrum of the west head signal. The values of the response function that were below the confidence level, were not considered in the analysis, and for the significant values the error bars were calculated (Bendat and Piersol, 2010).

Harmonic analysis was applied to the time series following Gomez-Valdes et al. (2012). Amplitudes and tidal phases of the main tidal constituents were obtained with the least squares method, which included the Rayleigh criterion of 1 and nodal corrections. The estimated uncertainty of the constituents was calculated with the signal-to-noise ratio (snr) parameter. The snr was calculated by \( \text{snr} = (M/\delta M)^2 \), where \( M \) is the amplitude of tidal constituents and \( \delta M \) is the amplitude error (Pawlowicz et al., 2002). When \( \text{snr} \) is high (\( \text{snr} \gg 1 \)), the harmonic constituent is well resolved, but when \( \text{snr} \) approaches 1 the constituent is unreliable because the uncertainty is of the same magnitude as the estimate.
Data from the current profiler were analyzed in the same way as those for sea surface elevation. Using the along channel velocities at the lagoon entrance, the power spectrum was calculated for each depth bin following Gomez-Valdes et al. (2012). The vertical structure of the currents was studied from the time average of the longitudinal velocity of each bin. The harmonic analysis of the currents was performed on the vertically averaged velocity and on each bin. In both cases, the tidal ellipse parameters (semi-major axis, semi-minor axis, phase, and orientation) were obtained following Pawlowicz et al. (2002).

4. Analytical model

4.1. Model description

The tidal circulation in a branched and choked lagoon was described with a three-dimensional, non-linear, homogeneous (constant water density) model as explained by Winant (2007). The linear, lowest-order solution of the model was used in the analysis. The model has been used in tidal propagation studies in gulfs (Winant, 2007) and in long estuaries (Waterhouse et al., 2011; Henrie and Valle-Levinson, 2014).

The model describes tidal wave propagation through semi-closed elongated water bodies using the relationship between geometric and frictional characteristics, assuming shallow water waves approximation, negligible lateral mixing, constant
vertical eddy viscosities, constant channel width and constant depth in the longitudinal axis. Hereafter dimensional variables are represented with asterisk.

The set of equations that describe the original non-linear problem for shallow elongated channels depended on five non-dimensional parameters $\varepsilon, \alpha, \delta, \kappa$, and $f$. The maximum depth of the system $H^*$ was considered much greater than the open-end tidal amplitude $C^* \varepsilon = C^*/H^* \ll 1$. The horizontal aspect ratio was $\alpha$, it was the ratio between width $B^*$ and length $l^*$ of channel, $\alpha = B^*/l^*$. For elongated basins, ($\alpha \ll 1$), the lower order solution predicts zero pressure gradient at the $y$-axis momentum equation. The frictional parameter $\delta$ relates friction to local acceleration (a proxy of the Stokes number) and it was defined as $\delta = (2K^*/\omega^*H^{*2})^{1/2}$, where $K^*$ stands for the vertical eddy diffusivity and $\omega^*$ for the tidal frequency. The geometrical parameter was $\kappa$. It represents the relative measure between the length of the channel and the wavelength of the tidal wave, it was defined as $\kappa = \omega^*l^*/(g^*H^*)^{1/2}$, where $g^*$ stands for gravitational acceleration. Finally, $f$ was the non-dimensional Coriolis parameter it was defined as $f = f^*/\omega^*$, where $f^*$ stands for the Coriolis frequency.

Because $\varepsilon$ was assumed to be small, the advective terms in the original momentum equations were neglected and the problem was broken into an ordered set of linear problems for the dependent variables. The resulting linear problem in the non-dimensional form to lowest order depended on four parameters $\alpha, \delta, \kappa$, and $f$; the linearized non-dimensional momentum equations were given by
\[
\frac{\partial u}{\partial t} - f \alpha v = - \frac{1}{\kappa^2} \frac{\partial \eta}{\partial x} + \frac{\delta^2}{2} \frac{\partial^2 u}{\partial z^2}, \quad (1)
\]

\[
\frac{\partial v}{\partial t} + \frac{f}{\alpha} u = - \frac{1}{\alpha^2 \kappa^2} \frac{\partial \eta}{\partial y} + \frac{\delta^2}{2} \frac{\partial^2 v}{\partial z^2}; \quad (2)
\]

and the continuity equation in the form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (3)
\]

Where the boundary conditions were no slip at the bottom \((z = -h)\) and kinematic and dynamic boundary conditions at the surface \((z = 0)\). The no-slip condition considered that the velocities were zero at the bottom (i.e. \(u = v = w = 0\)). At the surface, the kinematic condition implies that the water particle never leaves the surface (\(w = \frac{\partial \eta}{\partial t}\)). The dynamic condition indicates that the pressure is constant and there is no shear stress (i.e. \(\partial u / \partial z = \partial v / \partial z = 0\)). Since there was not lateral mixing in the model, there was no need for non-slip and shear stress boundary conditions at the lateral boundaries. On the open boundary the tide forces the sea level.

Using periodic solutions to find the corresponding set of differential equations from the vertically integrated continuity equation, Winant (2007) expanded the amplitude \((N)\) in function of powers of \(\alpha\) and found a lower order \((N^{(0)})\) solution for sea surface elevation that satisfies the boundary conditions. The solution was given by

\[
N^{(0)} = \frac{\cos[k \mu (1 - \chi)]}{\cos(k \mu)}. \quad (4)
\]
This solution was limited to the constant width and the depth on function of $y$-axis only. The frictional effects were represented by the parameter $\mu = \langle M_o \rangle^{-1/2}$, where the angled brackets show the cross-sectional average of $M_o$, and $x$ was the longitudinal position. In turn, $M_o$ was giving by

$$M_o = (f^2 Q_o^2 / P_0) - P_0,$$

(5)

where $P_0$ and $Q_0$ are complex functions of $(h, f, \delta)$. Depth was represented by $h$ and only depended on the $y$-axis ($y$ was the transverse-channel direction at all the zones). The $P_0$ and $Q_0$ equations were trigonometric complex functions of three terms; in both cases the first term was independent of $\delta$ (Winant, 2007). Him showed that when $\delta \to 0$, $M_o$ tended to $h$ and the real part of $\mu = \langle h \rangle^{-1/2}$ was independent of $f$. If $\delta \to \infty$, $M_o$ tended to $-P_0$ and also it was independent of $f$. Thus Winant (2007) found that the low order solution (equation 4) was practically not affected by rotation, which implies that the $f$ term in the momentum equation is negligible. Further information on calculating $P_0$, $Q_0$ and the $N^{(0)}$ lower order solution can be found in Winant (2007).

### 4.2. Analytical solution

For this analysis $h$ was represented by the function

$$h = 0.01 + 0.99(1 - y^4).$$

(6)

The bottom shape was selected to represent dredged navigational channels (especially Zones 1 and 3). However, following Winant (2007) and Waterhouse et al. (2011), other theoretical bathymetric shapes were tested and produced similar
results. Hence, the $N^{(0)}$ solution depended on two non-dimensional parameters: $\kappa$ and $\delta$. The methodology of Henrie and Valle-Levinson (2014) was applied to obtain the best $N^{(0)}$ solution using equation (4) for the system by solving for $\delta$ at each Zone with $K_1$ or $M_2$ tide frequencies. In the study area, the geometric characteristics of the channels were met with model requirements. The $\kappa$ parameters can be calculated using geometric dimensions at each channel and the frequency of the tide ($K_1$ or $M_2$). However, $\delta$ is undefined because the vertical eddy diffusivity is usually unknown. This method considers the wide range of $\delta$, from frictionless ($\delta < 1$) to highly frictional ($\delta \gg 1$). The $\kappa$ parameter was calculated and constant in each Zone because it depends on the geometric and tidal characteristics. However, this method requires also the range of $\kappa$ with the same length of $\delta$. $\kappa$ range should included the value of $\kappa$ calculated for each Zone. The range of combinations of $\kappa$ and $\delta$ parameters for each tidal frequency yielded a $N^{(0)}$ solution.

Around 4000 values of $N^{(0)}$ were calculated with equation (4). The number of solutions depends on how large the range of $\delta$ is (the length of the range $\delta$ was arbitrarily selected, but in all the cases was from $\delta < 1$ to $\delta \gg 1$). Each $N^{(0)}$ solution was compared to the amplitude of the principal harmonic of the observed tides at each station using the root mean square error. Therefore, the parametric space among $\delta$ (axis $x$), $\kappa$ (axis $y$) and minimum root mean square error (axis $z$) were generated for all stations and for diurnal and semidiurnal signals. Each parametric space contains a minimum root mean square error band. The minimum
root mean square error bands indicate the best solutions; the specific \( \delta \) values were selected using the intersection with \( \kappa \) values at each station. The minimum root mean square error solutions produced optimal values of \( \kappa \) and \( \delta \) for each selected frequency and zones. Hence \( \delta \) values do not require the direct solution of the vertical eddy diffusivity. Nonetheless, from \( \delta \) optimal values, the eddy viscosity can be calculated to each zone and frequency.

When frictional characteristics of the bodies of water are unknown, using this methodology it is possible to determine if the systems are highly frictional basins \((\delta > 1)\), slightly frictional \((\delta \approx 1)\) or frictionless \((\delta < 1)\). The last two having equation (4) as the solution (Winant, 2007). But in the highly frictional case \((\delta > 1)\), the dynamical balance describing the longitudinal propagation of the tidal signal is reduced to pressure gradient and frictional forces. For this situation the results of Henrie and Valle-Levinson’s (2014) methodology (optimal \( \kappa \) and \( \delta \) solutions from equation (4)) was an approach only used for diagnostics and classification. Knowing that this study site is highly frictional, and following Winant (2007), equation (4) was not used because the wave is damped towards the closed end and instead, the equation for tidal longitudinal propagation \((N^{(0)})\) in highly frictional water bodies was taken as

\[
N^{(0)} = \exp[-(1 + i)\kappa \delta \gamma x], \quad (7)
\]

where \( N^{(0)} \) is a complex function defined by \( \delta, \kappa, x, \) and \( \gamma \). The \( \gamma \) parameter depends on the geometry of the basin and is of order 1 (Winant, 2007). To select the optimal values of \( \gamma \) at each Zone, sensitivity analysis between 0.2 and 1.2 were
performed. The best fit between observations and the model results gave the optimum values of $\gamma$. For Zone 1 (short channel) lower values of $\gamma$ (~0.2) were used and for Zones 2 and 3 (long channels) midpoints values (~0.7) were used. The $N^{(0)}$ solution satisfies the 1-D diffusion equation, which is typical of highly frictional basins (LeBlond, 1978; Friedrichs, 2010). Hence, the solutions along the each one of the channels were expressed as

$$N_1^{(0)} = \exp[-(1 + i)\kappa_1\delta_1 y_1 x_1], \quad (8)$$

$$N_2^{(0)} = \exp[-(1 + i)\kappa_2\delta_2 y_2 x_2], \quad (9)$$

$$N_3^{(0)} = \exp[-(1 + i)\kappa_3\delta_3 y_3 x_3], \quad (10)$$

where 1, 2, 3 stand for the zones.

### 4.3. Channel connections

The Lighthill (1978)'s approach was used to obtain the spatial variations of the tidal signal along the entirety of each branch, without losing the $\delta$ and $\kappa$ characteristics of each channel. The $N^{(0)}$ solution is the non-dimensional attenuation ratio

$$N^{(0)} = \frac{\eta^*}{C^*}, \quad (11)$$

for $N_1^{(0)}$ at Zone 1, $C^*$ was the observed tidal harmonic amplitude (diurnal and semidiurnal in this case), therefore $\eta_1^* = (C^*)N_1^{(0)}$. The bifurcation area was located at $x = l$, where $l$ was the distance of the mouth to the bifurcation. The amplitude calculated at $x = l$ of Zone 1 was used at the beginning of Zones 2 and 3 in the
middle of the lagoon (bifurcation area, Fig. 1c) with \( N_1^{(0)} \) as a common factor of in both channels. Therefore, the solution along each channel was expressed as

\[
N_1^{(0)} = \frac{\eta_1^*}{C^*}, \quad (12)
\]

\[
N_2^{(0)} = \frac{\eta_2^*}{\eta_1^{*(x=l)}}, \quad (13)
\]

\[
N_3^{(0)} = \frac{\eta_3^*}{\eta_1^{*(x=l)}}. \quad (14)
\]

The model was forced at the mouth by the observed diurnal and semidiurnal tidal amplitudes. The solution curves of (12), (13) and (14) were normalized with the total length of each branch, such that \( x^*/L_{W,E}^* \), where \( L_W^* \) and \( L_E^* \) are the total length of west and east branch, respectively.

When the Coriolis term was negligible, the lower order longitudinal local velocity solution \( U \) was function of \( N_x^{(0)}, p_0, \) and \( k \), where \( p_0 \) was complex function of \( h, z \) and \( \delta \). The longitudinal local velocity was depth-integrated to obtain horizontal transport in the longitudinal direction and for specific Zone was given by

\[
[U]_j = \frac{i}{\kappa^2} \frac{\partial N_x^{(0)}}{\partial x} - M_0, \quad (15)
\]

where the \( j \) subscript indicates the solution Zone (1, 2 and 3) and is directly related to the solutions along each channel (1, 2 and 3, in equations (12), (13) and (14), respectively). The longitudinal transport \([U]\) was calculated for each channel using
in each \( \frac{\partial N_1^{(0)}}{\partial x}, \frac{\partial N_2^{(0)}}{\partial x} \) or \( \frac{\partial N_3^{(0)}}{\partial x} \); at the middle of the lagoon (closed end of Zone 1), \([U]\) satisfied the continuity condition (Lighthill, 1978).

5. Results

5.1. Sea surface elevation

The quality of the time series obtained from the pressure sensors (Fig. 2) inside the lagoon was verified by statistical comparison between the time series of the moored current profiler (used as a reference instrument) at the mouth (Fig. 5a) and the Diver CTD pressure signals throughout the lagoon. Because the Diver CTD sensors were operating correctly, the correlation between the two time series was close to 1. The sea surface elevation time series showed an attenuation and distortion of the tidal signal toward the heads (Fig. 2). The signals at C1 and C3 presented minimal distortion with respect to the inlet, with well-defined diurnal variations. However, semidiurnal variations during neap tides were distorted as broad and hooked waves. The C2 station was the shallowest and presented enhanced distortion at low water, especially in spring tide because during the lowest water levels the sensor was out of the water. The distortion observed in stations C1, C2 and C3 could be explained by friction effects in shallow water (LeBlond, 1978). In addition to the diurnal and semidiurnal oscillations observed in the time series, longer period fluctuations are visible. Atmospheric pulses were present in sea surface elevation variations as observed with the southerly...
Hurricane *Ernesto* winds, which caused a negative anomaly from 7 to 10 August 2012 (Fig. 2).

The most energetic tidal frequencies observed in the sea surface elevation were distinguished in the power spectra of the six time series (Fig. 3). The spectra showed three characteristic peaks: the diurnal band (~0.04 cph) was the most energetic, the low-frequency band (0.0042 to 0.0023 cph), and the semidiurnal band (~0.08 cph). Remarkably, the semidiurnal band was not statistically significant on the western side of the lagoon, unlike the rest. The time series of the west head was compared to that at C1 with the frequency response function. The gain diagram and phase diagram were obtained from the cross-spectra of sea surface elevation between station C1 and the west head (Fig. 4), and indicated that the causeway acted as a low-pass filter. Coherence was also calculated (not shown), the low frequency bands presented coherence values of around 0.6 and practically zero phase lags (Fig. 4, lower row), the rest of the bands showed lower coherence values than 0.6, therefore these bands were not significant. The response function analyses provided a quantitative explanation of the drastic attenuation and phase of the semidiurnal signal beyond the causeway. Another significant feature was the contribution of the third-diurnal band (~0.12 cph), especially in the middle of the lagoon. Furthermore, a low-frequency peak (~0.0027 cph equivalent to a period of 15 days) appeared in the entire lagoon. Mechanisms that could change the sea surface elevation with a periodicity of approximately 15 days were narrowed down to the wind (not discussed here) and a forced fortnightly tide in a frictional basin.
Kantha (2005) stated that the fortnightly variation in the Yucatan Shelf is modulated by the $K_1$ and $O_1$ interaction. Following Kinsman (1984), the modulation period ($T_{mod} = 4\pi/\Delta\sigma$, where $\Delta\sigma$ is the difference between $K_1$ and $O_1$ frequencies) was 13.7 days, and the modulation amplitude was 0.3 m. However, in the case of $M_2$ and $S_2$ the modulated period was 14.3 days and the modulation amplitude was 0.09 m. The frequency spectra were used to estimate the low-frequency amplitude at each station (Fig. 3). At zones 1 and 3 the low-frequency amplitudes were slightly amplified (from $\sim0.7$ m at the mouth to $\sim1.1$ m at east head), meanwhile at Zone 2 the signal was attenuated ($\sim0.45$ m) by the causeway effect. Even considering this attenuation, the differences between the observed and predicted low-frequency modulations were important at all sites. This suggested that low frequency sea level modulation might not only be caused by astronomic influence.

5.2. Currents

A harmonic analysis was performed to the along-channel velocities observed at the mouth (Fig. 5b) to determine amplitude, phase, and percentage of explained variance for the tidal components (Pawlowicz et al., 2002). Power spectra were calculated for the vertically averaged longitudinal velocity (Fig. 5c) and for each depth of the longitudinal velocity (not shown). The diurnal band was the most energetic throughout the water column, and maximum at the surface. The second most important energy peak appeared in the semidiurnal band, followed by third-diurnal energy that illustrated non-linear effects on the tidal flows (Fig. 5c).
The main tidal harmonics of the currents were explored with a tidal ellipse analysis (Table 1). These results were obtained with the vertically averaged velocity. The diurnal $K_1$-current and $O_1$-current showed amplitudes of $23.2 \pm 1.9$ and $25.3 \pm 2.4$ cm/s, respectively, with the $O_1$-current exhibiting less snr than the $K_1$-current. The semidiurnal principal components were $M_2$-current and $N_2$-current ($12.1 \pm 0.8$ and $3.5 \pm 0.8$ cm/s, respectively); with the lowest snr observed for the $N_2$-current. The other semidiurnal constituent, $S_2$, was considered insignificant inside the lagoon because its amplitude was the smallest at the mouth ($2.0 \pm 0.9$ cm/s). The semi-minor axis was small compared with the semi-major axis (ellipticity was $\sim 0.05$), which indicated quasi-rectilinear tidal flows (elongated ellipses); it was due to the narrow Chelem mouth. The diurnal ellipses rotated counterclockwise because their semi-minor axis was positive. The $M_2$ rotated clockwise because its semi-minor axis was negative, however this result was not reliable because its associated error was of similar magnitude and the measured amplitude of the axis was of the same order of magnitude to the instrument's accuracy ($\pm 0.5$ cm/s). The orientations of all the ellipses were consistent with the lagoon entrance, close to 90° counterclockwise from the east.

Tidal current ellipses were also calculated for each bin depth to obtain the vertical structure (Fig. 6). The magnitudes of the semi-major and semi-minor axes for the main diurnal components ($O_1$ and $K_1$) were similar when considering the error, but twice as large as that of the semidiurnal $M_2$ (Table 1). The vertical structures of the
magnitude and the phase for the semi-minor axes were practically homogeneous throughout the water column (Fig. 6 b, d). However, the vertical structures of the semi-major axes (Fig. 6a) in the tidal current components showed parabolic behavior (decreasing towards the bottom), because of bottom friction (MacCready and Geyer, 2010).

5.3. Tidal spatial variations

The harmonic analysis of the sea surface elevation yielded the main components for the diurnal and semidiurnal frequencies and only one significant compound tide (Table 2). The main diurnal components were $K_1$ and $O_1$ with amplitudes of $17.8 \pm 1.4$ and $17.3 \pm 1.8$ cm, respectively. The main semidiurnal harmonics were $M_2$ and $N_2$ with amplitudes of $5.7 \pm 0.3$ and $2.0 \pm 0.2$ cm, respectively. The semidiurnal harmonic $S_2$ had an amplitude of $1.1 \pm 0.3$ cm at the mouth, but was insignificant toward the west and the east ends.

The system appeared to be dominated by diurnal tides. The type of tide was characterized by the form number ($F$), defined as $F = (K_1 + O_1) / (M_2 + S_2)$ (Defant, 1958). The form number was between 5 and 8 for all sites, which indicate that the entire lagoon is diurnal dominant. The highest $F$ value was found in the west head where the differences between diurnal and semidiurnal components were greatest. The tidal range was approximately 0.7 m at the mouth and decreased toward the heads at all sites except at the west head. On the other hand, the fortnightly declinational variation ($\left|\frac{(K_1 + O_1)}{(K_1 - O_1)}\right|$) was larger than
the fortnightly synodical variation $|(M_2 + S_2)/(M_2 - S_2)|$. At Zone 1, the fortnightly declinational variations were two orders of magnitude larger than the fortnightly synodical variation. This difference was more pronounced toward the lagoon heads where the amplitudes became minimal. The maximum difference between these fortnightly variations was found at the mouth where the difference between the two amplitudes was largest.

None of the overtides was significant. The $MK_3$ compound tide was generated by the non-linear $K_1$-$M_2$ interactions and was observed in the entire lagoon, in particular at Zone 3, where it reached amplitude of $1.4 \pm 0.7$ cm (Table 2). The attenuation of $MK_3$ amplitude at Zone 2 was caused by the strong semidiurnal signal attenuation and small $K_1$-$M_2$ interaction. In systems dominated by diurnal tides, shallow water tidal components are normally generated in the third-diurnal band ($\sim 0.125$ cph) (Dworak and Gomez-Valdes, 2005). According to the analytical solutions for the interactions between two-tide components ($K_1$ and $M_2$) proposed by Parker (1991), the quadratic part of the nonlinear friction term, $u|u|/h$, accounts for the shallow water tidal components. A scaling analysis was performed between the nonlinear terms (the advective $u \partial u/\partial x$ and the frictional term $u|u|/h$) to demonstrate their relative importance. The magnitude of the tidal velocities ($u$) was of order $10^{-1}$ with a longitudinal scale, $x$, of order $1 \times 10^3$ and depth ($h$) of order 1. Therefore the magnitude of $1/h\ u|u|$ was at least two orders of magnitude larger than the other term of the one-dimensional momentum equation $(1/h\ u|u| \gg$
\( u \partial u / \partial x \). Furthermore, since the study area is a highly frictional lagoon, the balance between the pressure gradient and frictional forces controls it. Following Hill (1994), the balance was written as
\[
g \frac{\eta_0 - \eta_L}{L} = \frac{C_f u |u|}{(H + \eta_m)},
\]
where the sea surface elevation in the open end was \( \eta_0 \), the sea surface elevation in the lagoon was \( \eta_L \), \( \eta_m \) was the mean surface elevation between them, \( C_f \) was the dimensionless friction coefficient. Consequently the quadratic part of the non-linear friction term \( 1/h u |u| \) was expected as shallow water tidal generator.

The non-linear distortion between the \( M_2 \) and \( M_4 \) components is frequently used to determine the dominance between flood and ebb (Friedrichs and Aubrey, 1988). However, Ranasinghe and Pattiaratchi (2000) showed that this criterion does not apply to lagoons with diurnal dominance. Since the studied area is considered a shallow system, the method proposed by Friedrichs and Aubrey (1988) was applied. The ebb-flood asymmetry can be understood by analyzing the behavior of the tidal phase speed as a function of changes in width and depth of the basin. When a deep basin system is dominated by tidal variations, high tide propagates faster and partially catches up with the previous low tide. This situation produces a shorter rising tide and flood dominance. In the case of a wide basin system, low tide propagated faster partially catching up with the previous high tide, therefore, produces a shorter falling tide and ebb dominance. Friedrichs and Aubrey (1988) proposed ebb and flood-dominant areas at the parametric space between \( (a/\langle h \rangle) \) and \( (V_s/V_c) \). The first parameter is the ratio between tidal amplitude and the average channel depth. The second parameter is the ratio between the volume of
storage in intertidal zones $V_s$ and the volume of the channels $V_c$. In Chelem, these relations are small in all three zones ($a/\langle h \rangle \sim 0.1$) and $(V_s/V_c \sim 0.1)$, because the average depth of all zones is small but larger than the tidal amplitude. The combination of the two ratios classifies the system as ebb-dominant.

The harmonic analysis was also used to estimate tidal attenuation and phase of the sea surface elevation (Table 2). Diurnal signal attenuations were greatest at the end of Zones 2 and 3, and the time delay observed were of ~8 h and ~3 h at the west and east heads, respectively. The maximum semidiurnal attenuation was ~5 and ~4 cm with a time delay of ~5 and ~3 h at the end of Zones 2 and 3, respectively (Table 3). In Zone 2, the semidiurnal signal was practically filtered out. Our results indicated that frictional effects were the main attenuators of the tides. This was corroborated by the application of an analytical model that required frictional effects to resemble observations, as explained next.

5.4. Model results

Following Henrie and Valle-Levinson (2014), Winant (2007)'s model was applied piecewise to Chelem. This required calculation of the $\delta$ and $\kappa$ parameters for each zone (Fig. 7a; Table 3). Values of $\delta > 1$ throughout the lagoon indicated that the lagoon was highly frictional; therefore, (7) was the solution used. Zones 1 and 3 exhibited the greatest $\delta$ and smallest $\kappa$ values. Zone 1 also showed the least amplitude attenuation. Zone 2 showed the lowest $\delta$ and the highest $\kappa$ values.
because of its dimensions. From $\delta$ optimal values, the eddy viscosities were calculated to each zone and frequency. The eddy viscosity was approximately the same for diurnal and semidiurnal frequency, but it was varied at each zone. The vertical eddy viscosities were approximately 0.02 m$^2$/s at Zones 1 and 3, this value was similar to that found by Munk (1966). At the Zone 2 the vertical eddy viscosities were approximately 0.003 m$^2$/s, coinciding with the values reported by Waterhouse et al. (2011) for these type of channels.

Once the $\delta$ and $\kappa$ parameters were determined for each zone and for each tidal harmonic of interest. The Figure 7a shows the $\delta$ and $\kappa$ parametric space for the diurnal frequency in Zone 2 as an example, but the same analysis was performed for each zone and frequency. In this figure, the optimal $\delta$ value at the intersection of minimum root mean square error band and $\kappa$ value was represented with the white line (Fig. 7a). The model of Winant (2007) effectively described the tidal wave attenuation at each station (Fig. 7b). Tidal signal attenuation was observed at all stations inside the lagoon and the attenuation magnitude increased toward the west and east heads (Table 3). Tidal phases were calculated with the model and the phase differences with respect to the principal tidal components of the inlet were listed on Table 3. The analytical time delay diverged from the time delay observed (around 3.2 h) at the end of Zone 2 (west head) especially with the diurnal signal, while those of Zone 1 (0.1 h) and Zone 3 (0.6 h) were consistent (Fig. 7c). The analytical model describes the linear problem, the study area was highly frictional and the balance between the pressure gradient and frictional forces
controlled it. The fit between observations and the model results was suitable except in the causeway zone where advection terms may become relevant because of the narrow channel effects. The model limitations were more noticeable at Zone 2 especially in the phase differences between observations and model results.

The next step was to calculate the spatial variations of the longitudinal transport and phase within each zone for the semidiurnal (Fig. 8) and diurnal signals (Fig. 9). To verify model results, tidal transport observations at the inlet were compared to model results at Zone 1, using an approximate cross sectional area of 450 m². Analytical results were consistent with the observations. The observed diurnal (semidiurnal) tidal transport was ~110 m³/s (~50 m³/s) and the maximum value of the analytical result was ~100 m³/s (~40 m³/s). Analytical results showed that ~35% of the diurnal longitudinal transport goes to Zone 2, and ~65% to Zone 3. Similar results were obtained for the semidiurnal longitudinal transport (~30% to Zone 2 and ~70% to Zone 3). These differences in each zone were because δ and κ differ in each zone. The semidiurnal signal in the longitudinal transport was the most attenuated toward the heads: ~0.9 and ~5 m³/s at Zones 2 and 3, respectively. Time delay between the east head and both the end of Zone 1 and the mouth was ~2.5 and ~2.7 h, respectively (Fig. 8). The diurnal signal was also attenuated but it reached the closed ends of Zones 2 and 3 as ~10 and ~20 m³/s, respectively. The time delay in the diurnal transport toward Zones 2 and 3 was 4.3 and 3.5 h, respectively (Fig. 9).
6. Discussion

The tidal signal in the Yucatan Shelf is mainly diurnal as previously described by Kantha (2005) and Kjerfve (1981). Present results showed that the tidal signal in Chelem is also diurnal but with a slight semidiurnal influence. The principal diurnal components were $K_1$ and $O_1$ and the principal semidiurnal components were $M_2$ and $N_2$. The semidiurnal component $S_2$ was small in the lagoon because $M_2$ and $S_2$ amphidromic points are located in the middle of the Yucatan Shelf, and the region for $S_2$ minimum amplitude is larger than for $M_2$ (Kantha, 2005). The tidal-ellipses from the lagoon entrance were consistent with the harmonic analysis results using the sea surface elevation data and with previous reports of tides in the GoM. David and Kjerfve (1998) reported that at the inlets of the Terminos lagoon (located ~400 km southwest of Chelem) the $K_1$-current and $O_1$-current were the main diurnal components and the $M_2$-current was the main semidiurnal component. The magnitudes reported in these cases were similar to those obtained in Chelem.

At the Chelem inlet, the phase difference between $K_1$ water level and the $K_1$-current was $\sim49^\circ$ (Table 1 and 2). This behavior is typical of highly frictional regions and is consistent at the Chelem inlet. In this situation, the current and sea surface elevation phase difference should be $45^\circ$ (Winant, 2007). Friedrichs (2010) showed that in long, shallow, non-convergent estuaries friction dominates acceleration in the momentum balance. Therefore, the dynamics has the form of a 1-D diffusion equation.
Although this work did not intend to classify the lagoon morphologically, the behavior of the west channel (beyond the causeway) corresponded to a choked system as described by Kjerfve and Magill (1989). The tidal signal in Chelem reached the western branch as it passed through two gates in the causeway. In this process, frequencies higher than the diurnal signal were filtered out. Kjerfve and Magill (1989) quantified the filtering of $K_1$ and $M_2$ harmonics in choked systems using the coefficient of repletion ($R$). This coefficient provided a quantitative measure of filtering characteristics at the inlet channels and was calculated as

$$R = \left( \frac{T}{2\pi a_0} \right) \left( \frac{A_c}{A_B} \right) \left[ 2ga_0/\left(1 + 2gLn^2r^{-4/3}\right) \right]^{1/2}, \quad (16)$$

where the tidal period is $T$, the tidal range in the adjacent body of water is $2a_0$, $A_c/A_B$ is the relation between the cross-sectional channel area and the surface basin area, the gravity is $g$, the length of the entrance is $L$, the hydraulic radius of the channel is $r$, and $n$ is the Manning’s channel friction range (between 0.01 and 0.10 s/m$^{1/3}$). For the analysis of the Chelem causeway effect, the tidal periods ($T$) were $K_1$ and $M_2$, the tidal range at the Zone 1 (adjacent body of water for this case) was 0.8 m, the relation between the cross-sectional channel area and the surface basin area at Zone 2 was $A_c/A_B \sim 5.9 \times 10^{-6}$, the gravity is $g$ (9.8 m/s$^2$), the length of the entrance was $L \sim 20$ m, the hydraulic radius of the channel was $r \approx 1.75$ m (rectangular shape channel was considered), and Manning’s friction range at the channel was taken as 0.10 s/m$^{1/3}$. The non-dimensional coefficients of repletion were calculated for both $K_1$ and $M_2$ harmonics as 0.24 and 0.12, respectively, corresponding to high choking regimes (Hill, 1994). A frequency response function
analysis between the C1 signal and the west head signal confirmed such filtering. The semidiurnal amplitude of 0.5 cm obtained from the harmonic analysis was of the same magnitude as the resolution of the instruments and therefore could not be properly measured.

Geometric parameters determine spatial variations of coastal tides (Friedrichs, 2010). The relative importance of the geometric characteristics was represented by the \( \kappa \) parameter, which was >1 throughout the lagoon. In fact, the diurnal tidal wavelength was two orders of magnitude longer than the basin length (~11.5 km between the inlet and the end of Zone 2), and more so with the shorter basins at Zones 1 and 3. Each zone had different \( \kappa \) values, related to their specific geometric characteristics. Zones 1 and 3 had similar \( \kappa \) values (0.05 and 0.08, respectively) for diurnal and (0.10 and 0.15, respectively) for semidiurnal signals. Both the diurnal and semidiurnal wavelengths were longer than the combined length of Zones 1, 2 and 3.

According to Friedrichs (2010), a tidal signal becomes altered by deformation and attenuation because of frictional effects when it enters a water body. Both phenomena (deformation and attenuation) are observed from the mouth toward the heads (Fig. 2). In highly frictional lagoons, local acceleration and Coriolis forces are considered insignificant (Winant, 2007). Consequently, the balance between pressure gradient and friction determines tidal propagation. Our results showed that Chelem is a highly frictional lagoon because the frictional parameter \( \delta > 1 \) for
both the diurnal and semidiurnal signals in all zones. Frictionally, Zones 1 and 3 had close \( \delta \) values for both diurnal (15.7 and 14.7 respectively) and semidiurnal signals (8.4 and 9.2 respectively). The diurnal and semidiurnal frictional parameters at Zone 2 were 9.4 and 7.3, respectively.

Geometric characteristics at Zone 2 produced the highest values of the geometrical parameter; these high values increased the relative importance of the pressure gradient in the highly frictional momentum balance. This compensation (pressure gradient dominated) between the pressure gradient and geometric characteristics is greater at Zone 2 (especially in the semidiurnal case) and the frictional parameter at Zone 2 was the lowest.

Consequently, the results of the sea surface elevation analysis and longitudinal transport demonstrated that Zone 2 had the greatest attenuations because of a physical restriction (causeway), even though it displayed the least friction. Theoretically the model was not sensitive to the causeway, with the solution based on the geometric and frictional parameters (Winant, 2007). However, frictional parameters were optimized by reducing the root mean square error between model results and observations (Henrie and Valle-Levinson, 2014). Therefore, the estimated \( \delta \) values included the frictional effects evident in the observations that could involve for example, physical restrictions and changes in bottom roughness.
In addition to the observed tidal frequencies, low frequency variability (~15-day period) was present in the entire lagoon. These low frequency signals could be generated by the non-lineal interactions of the major tidal harmonics (beats or modulated frequency). However, fortnightly variations in the tides can be influenced by bottom friction, as has been regularly observed in highly frictional semi-enclosed basins with narrow and shallow inlets connected to the ocean. This effect is caused by increased friction and pressure gradients that produce higher lagoon water levels. The intensity of this phenomenon depends on the spring and neap tide cycles, where spring tides would intensify the effect while neap tides would minimize it (LeBlond, 1979; Hill, 1994). As the Chelem fortnightly declinational variations were larger than the fortnightly synodical, the most likely synchronization of the spring-neap cycles is with the 27.32 days orbital cycle of the Moon (tropical month). Another factor that contributes to low frequency oscillations is the wind. There is a well-supported hypothesis about the influence of the E-W wind stress on subtidal variations in lagoons (Gutiérrez de Velasco and Winant, 2004), but this effect has not been studied in Chelem. The effect of winds on subtidal variations should be examined in a future study.

7. Conclusions

This study showed that tidal currents inside a shallow semi-enclosed branched system were driven by the barotropic pressure gradient force acting over a short distance equivalent to ~1/10 the tidal wavelength. The effects of a causeway were
accounted for as frictional and geometric parameters were optimized according to observations. *Chelem* was determined to be a highly frictional coastal body of water where pressure gradient and frictional forces control the hydrodynamics. The tidal signal was strongly attenuated as it propagated within the lagoon because the geometric characteristics of the lagoon contributed to high friction. The tidal signal was most attenuated toward the west head as the causeway acted as a hydraulic low-pass filter. This illustrated the effects of anthropogenic modifications to coastal lagoon systems, which clearly alter tidal dynamics.

**Acknowledgments**

We are grateful to Gilberto Jeronimo Moreno for providing the data and to CONACYT’s M0023- FOMIX YUCATAN project for supporting this study. We also thank CONACYT for granting scholarship No. 206018 to Leonardo Tenorio-Fernandez. AVL acknowledges support of NSF project OCE-1332718. We also acknowledge the critical comments from the reviewers.

**References**


**Figure captions**

Figure 1. - a) Location of *Chelem* in the Yucatan Peninsula, Mexico. b) *Chelem* sampling locations with the black dots representing the locations of moored Diver CTD, a black ring represented the location of the current profiler (Aquadopp) and Diver CTD moored at the mouth, gray contours show approximate depth. At the left hand corner is represented the channel network. c) Analytical model setup, showing Zones 1, 2 and 3, with the x axis representing the along-channel direction within all the zones.

Figure 2. - Time series of the sea surface elevations in cm for all measurement stations spanning from 27 June 2012 to 25 August 2012.

Figure 3. - Power spectra estimations normalized by the mean for all measurement stations using the sea level elevation data obtained from a 60-day time series the (black line and crosses). The gray band represents the confidence interval of each set of data and the black continuous line the 95% confidence level.

Figure 4. - Response function between the cross-spectral density of sea surface elevation at station C1 and the west head. Upper row: gain diagram, the vertical lines are the error bar of gain values. Lower row: phase diagram. In both diagrams the crosses indicate the intersection between the frequency and the gain or phase values.
Figure 5. - (a) Sea surface elevation measured with the Aquadopp-sensor of pressure spanning from 27 June 2012 to 25 August 2012. (b) Longitudinal velocity at each depth yielded at the inlet by current profiler measurements, same time span. Positive velocities represent inflow and negative velocities represent outflow. (c) Power spectrum (PSD) in \((\text{m/s})^2/\text{cph}\) of the depth-averaged longitudinal velocity.

Figure 6. - Vertical structure of the ellipse tidal current parameters: the amplitude of the a) semi-major and b) semi-minor axis (cm/s), c) orientation in degrees of the semi-major axis relative to the East, and d) the phase in degrees of the tidal potential relative to Greenwich.

Figure 7. - (a) Example of optimal \(\kappa\) and \(\delta\) values obtained by root mean square error analysis of observational data and analytical solutions for the diurnal frequency in Zone 2. (b) Comparison of the tidal amplitude attenuation of observational data with analytical solutions (\(N^0\)) of diurnal and semidiurnal tides in all three zones of Chelem, using the solution for \(\delta>1\). (c) The corresponding phase difference for the tidal amplitude attenuations of (b). In both (b) and (c), the \(x\)-axis are normalized with the total length \(L\) of the west and east branch (beginning at the mouth). The western and the eastern channels are represented with blue and blacks lines, respectively. The solid lines are the diurnal signal and the dashed lines are the semidiurnal signal. At the beginning and at the ending of each zone the observations are represented by circles for the semidiurnal signal, and by crosses for the diurnal signal.
Figure 8. - Longitudinal transport $[U]$ of the semidiurnal tide ($M_2$) in $m^3/s$. White lines represent the phase for a) Zone 3 (eastern side), b) Zone 1 (central part), and c) Zone 2 (western side).

Figure 9. - The same as Fig. 8, for the diurnal tide ($K_1$).

**Legends to tables**

Table 1: Top section: geographic location of the measurement stations and average dimensions of each zone: length ($l$), width ($W$), and depth ($H$). Lower section: ellipse parameters of the most important tidal currents for the vertically averaged velocity at the inlet of Chelem. $M$ is the amplitude of the semi-major axis in cm/s, $m$ is the amplitude of the semi-minor one in cm/s, $\phi$ is the inclination in degrees, and $\theta$ is the phase in degrees. The error of each parameter is denoted by $\delta M$, $\delta m$, $\delta \phi$, and $\delta \theta$; and $snr$ indicates the signal-to-noise ratio.

Table 2: Principal tidal constituents and compound tide extracted from sea surface elevation data obtained in a period of 60 days from Chelem measurement stations: Mouth ($M$), C1, West Head (WH), C2, C3, and East Head (EH). Amplitude ($A$) in cm, phase ($\theta$) in degrees, amplitude error ($\delta A$) in cm and phase error ($\delta \theta$) in degrees. Values less than 1 cm were excluded.
Table 3: Observations and analytical model results of the sea surface elevation attenuation and time delay of diurnal and semidiurnal signals at the three zones. The form numbers \((F)\) were estimated using the observations; \(\kappa\) and \(\delta\) are the optimal values for each zone; \(\eta_i/\eta_i\) is the attenuation fraction relative to the inlet for both signals at each zone; \(\theta_c - \theta_i\) is the signal time delay in hours relative to the inlet. \(\eta_i\) and \(\theta_i\) are the sea surface elevation amplitude and phase at the mouth, and \(\eta_c\) and \(\theta_c\) are amplitude and phase at the closed end of each zone.
Figure
### Table

<table>
<thead>
<tr>
<th>Zone</th>
<th>Station name</th>
<th>Geographic location Lat. N°/ Long. W°</th>
<th>Dimensions</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<tr>
<td></td>
<td>C1</td>
<td>21°15'44.66&quot;/89°42'11.49&quot;</td>
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<tr>
<td>2</td>
<td>West Head</td>
<td>21°14'32.22&quot;/89°46'55.49&quot;</td>
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<tr>
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<td>C2</td>
<td>21°16'01.18&quot;/89°40'36.51&quot;</td>
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<td>East Head</td>
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#### Tidal currents at the inlet

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<tr>
<th>Cons.</th>
<th>Freq. (cph)</th>
<th>M (cm/s)</th>
<th>δM (cm/s)</th>
<th>m (cm/s)</th>
<th>δm (cm/s)</th>
<th>φ (°)</th>
<th>δφ (°)</th>
<th>θ (°)</th>
<th>δθ (°)</th>
<th>Snr</th>
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<td>K1</td>
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<td>1.9</td>
<td>1.1</td>
<td>0.6</td>
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<td>O1</td>
<td>0.03873</td>
<td>25.3</td>
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<td>M2</td>
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<td>0.8</td>
<td>-0.6</td>
<td>0.5</td>
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<td>7.9</td>
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Table 2

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<th>Semidiurnal</th>
<th>Compound tide</th>
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<tr>
<td></td>
<td>$K_1$ (0.0417 cph)</td>
<td>$O_1$ (0.0387 cph)</td>
<td>$M_2$ (0.0805 cph)</td>
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<tr>
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<td>$A$ (cm)/$\theta^\circ$</td>
<td>$\delta A/ \delta \theta$</td>
<td>$\delta A/ \delta \theta$</td>
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<tr>
<td>1</td>
<td>M</td>
<td>17.8/336</td>
<td>1.4/5</td>
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<td>1.0/10</td>
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<td>12.8/347</td>
<td>1.4/7</td>
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<td>EH</td>
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<td>1.0/7</td>
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Table 2
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<th>$\kappa$</th>
<th>$\delta$</th>
<th>$\eta_c/\eta_i$</th>
<th>$\theta_c - \theta_i$ (h)</th>
<th>$\kappa$</th>
<th>$\delta$</th>
<th>$\eta_c/\eta_i$</th>
<th>$\theta_c - \theta_i$ (h)</th>
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<td>Zone 1 (Observation)</td>
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<td>0.5</td>
<td>0.86</td>
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Table 3